7. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity.

- the velocity of A is $40\,\text{km}\,\text{h}^{-1}$ due east
- the velocity of B is $20\,\text{km}\,\text{h}^{-1}$ on a bearing of angle α ($0° < \alpha < 90°$), where $\tan\alpha = \frac{4}{3}$

The boats are modelled as particles.

(a) Find, in terms of **i** and **j**, the velocity of B in $\text{km}\,\text{h}^{-1}$

(2)

At noon

- the position vector of A is $20\mathbf{j}\,\text{km}$
- the position vector of B is $(10\mathbf{i} + 5\mathbf{j})\,\text{km}$

At time t hours after noon

- the position vector of A is $\mathbf{r}\,\text{km}$, where $\mathbf{r} = 20\mathbf{j} + 40t\mathbf{i}$
- the position vector of B is $\mathbf{s}\,\text{km}$

(b) Find an expression for **s** in terms of t, **i** and **j**.

(2)

(c) Show that at time t hours after noon,

$$\overrightarrow{AB} = [(10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}]\,\text{km}$$

(2)

(d) Show that the boats will never collide.

(3)

(e) Find the distance between the boats when the bearing of B from A is 225°

(4)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 13 marks)

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