8. (a) Use integration to show that

$$\int x\cos 2x\,\mathrm{d}x = Ax\sin 2x + B\cos 2x + c$$

where A and B are constants to be found, and c is an arbitrary constant.

(3)

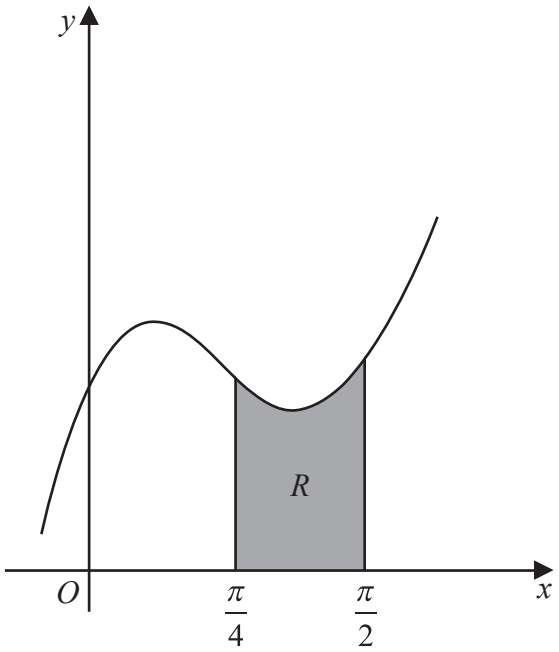


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 4x + 3\cos 2x$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = \frac{\pi}{2}$, the x-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution S.

(b) Use algebraic integration to find the exact volume of S, giving the answer in the form

$$\pi\left(p\pi^3 + q\pi + r\right)$$

where p, q and r are constants.

(6)

(Total for Question 8 is 9 marks)

9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A drink is put in a freezer to cool.

The temperature, $\theta°\text{C}$, of the drink, t minutes after it is put in the freezer, is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta + 15)$$

where k is a positive constant.

Given that the temperature of the drink when it was put in the freezer was $21°\text{C}$,

(a) solve the differential equation to show that, according to the model,

$$\theta = A\mathrm{e}^{-kt} + B$$

where A and B are integers to be found.

(5)

Given that 20 minutes after the drink is put in the freezer, its temperature is $12.9°\text{C}$,

(b) find the value of k, giving the answer to 3 significant figures.

(2)

(c) Hence find, according to the model, the temperature of the drink, one hour after it was put in the freezer. Give the answer to one decimal place.

(2)

(Total for Question 9 is 9 marks)