7. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
- the point B has position vector $3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- the point C has position vector $\alpha\mathbf{i} + \beta\mathbf{j} - 5\mathbf{k}$

where α and β are constants.

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

Given that C lies on l

(b) find the value of α and the value of β.

(2)

(c) Find, in degrees to one decimal place, the acute angle between OC and AB.

(3)

The point D lies on l such that OD is perpendicular to l.

(d) Find the exact coordinates of D.

(3)

(Total for Question 7 is 10 marks)

8. (a) Use integration to show that

$$\int x\cos 2x\,\mathrm{d}x = Ax\sin 2x + B\cos 2x + c$$

where A and B are constants to be found, and c is an arbitrary constant.

(3)

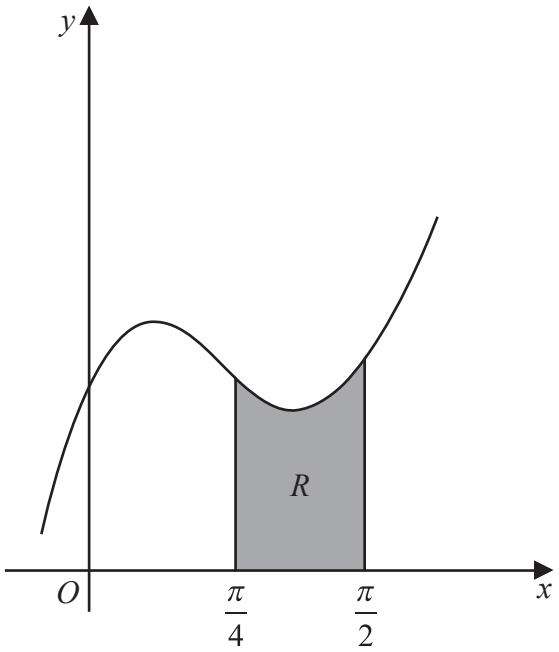

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 4x + 3\cos 2x$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = \frac{\pi}{2}$, the x-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution S.

(b) Use algebraic integration to find the exact volume of S, giving the answer in the form

$$\pi\left(p\pi^3 + q\pi + r\right)$$

where p, q and r are constants.

(6)

(Total for Question 8 is 9 marks)