4. The curve C has parametric equations

$$x = \frac{3}{5t-2} \qquad y = \frac{t}{4-t} \qquad t \in \mathbb{R} \qquad t \neq \frac{2}{5} \qquad t \neq 4$$

(a) Show that the Cartesian equation of C is

$$y = \frac{ax+b}{cx+d} \qquad x \neq -\frac{d}{c}$$

where a, b, c and d are integers to be found.

(3)

The line l has equation $y = -x + 1$

(b) Use algebra to prove by contradiction that l and C do not intersect.

(4)

(Total for Question 4 is 7 marks)

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5. Use the substitution $u = \sqrt{2x-1}$ to show that

$$\int \frac{x}{\sqrt{2x-1}}\,\mathrm{d}x = A(2x-1)^{\frac{1}{2}}(x+B) + k$$

where A and B are constants to be found, and k is an arbitrary constant.

(5)

(Total for Question 5 is 5 marks)

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6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$x^2 - xy + y^2 + 4x = 12$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(4)

Given that the point $P(-4, -6)$ lies on C,

(b) find the equation of the normal to C at P, giving the answer in the form $y = mx + c$ where m and c are constants.

(3)

The normal to C at P intersects C again at the point Q.

(c) Find the exact coordinates of Q.

(3)

(Total for Question 6 is 10 marks)

7. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
- the point B has position vector $3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- the point C has position vector $\alpha\mathbf{i} + \beta\mathbf{j} - 5\mathbf{k}$

where α and β are constants.

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

Given that C lies on l

(b) find the value of α and the value of β.

(2)

(c) Find, in degrees to one decimal place, the acute angle between OC and AB.

(3)

The point D lies on l such that OD is perpendicular to l.

(d) Find the exact coordinates of D.

(3)

(Total for Question 7 is 10 marks)