10.

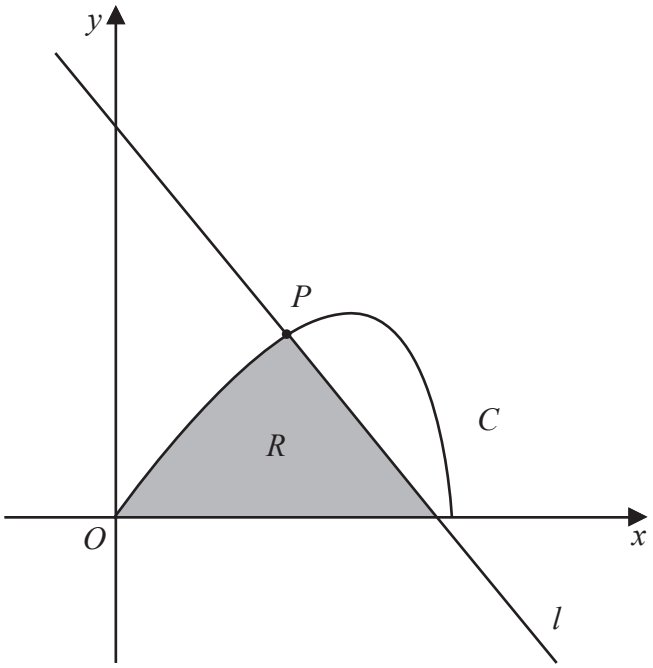


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 8\cos\theta \qquad y = 5\sin 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The point P with parameter $\theta = \frac{\pi}{3}$ lies on C.

The line l is the normal to C at P, as shown in Figure 2.

(a) Use calculus to show that l has equation

$$y = p\sqrt{3}\,x + q\sqrt{3}$$

where p and q are rational constants to be found.

(4)

The finite region R, shown shaded in Figure 2, is bounded by C, l and the x-axis.

(b) Using algebraic integration, find the exact area of R, giving the answer in the form $A + B\sqrt{3}$ where A and B are rational constants.

(6)

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS