Question Number	Scheme	Marks
7 (i)	Way One: $\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x = \frac{1}{4}\mathrm{e}^{2x}\sin 4x - \int \frac{1}{2}\mathrm{e}^{2x}\sin 4x\,\mathrm{d}x$	M1, A1
	$= \frac{1}{4}\mathrm{e}^{2x}\sin 4x - \left(-\frac{1}{8}\mathrm{e}^{2x}\cos 4x + \int \frac{1}{4}\mathrm{e}^{2x}\cos 4x\,\mathrm{d}x\right)$	dM1, A1
	$\frac{5}{4}\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x = \frac{1}{4}\mathrm{e}^{2x}\sin 4x + \frac{1}{8}\mathrm{e}^{2x}\cos 4x$	ddM1
	$\Rightarrow \int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x = \frac{1}{5}\mathrm{e}^{2x}\sin 4x + \frac{1}{10}\mathrm{e}^{2x}\cos 4x + c$	A1
		(6)
(ii)	$2x = \sin u \Rightarrow 2\mathrm{d}x = \cos u\,\mathrm{d}u$	
	$\int \frac{2x^3}{\sqrt{1-4x^2}}\,\mathrm{d}x = \int \frac{1}{8}\frac{\sin^3 u}{\sqrt{1-\sin^2 u}} \times \cos u\,\mathrm{d}u$	M1
	$= \int \frac{1}{8}\sin^3 u\,\mathrm{d}u$	A1
	$= \int \frac{1}{8}\sin u\left(1-\cos^2 u\right)\mathrm{d}u = \int \frac{1}{8}\sin u - \frac{1}{8}\sin u\cos^2 u\,\mathrm{d}u$	dM1
	$= \left[-\frac{1}{8}\cos u + \frac{1}{24}\cos^3 u\right]$	ddM1 A1
	States or uses limits $\frac{\pi}{6}$ and $\frac{\pi}{3}$ E.g. $\left(-\frac{1}{8}\cos\frac{\pi}{3} + \frac{1}{24}\cos^3\frac{\pi}{3}\right) - \left(-\frac{1}{8}\cos\frac{\pi}{6} + \frac{1}{24}\cos^3\frac{\pi}{6}\right)$	M1
	$= \frac{3}{64}\sqrt{3} - \frac{11}{192}$	A1
		(7)
		(13 marks)

(i) Condone missing dx's throughout

M1: Integrates by parts to a correct form. E.g.

$$\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x = \pm A\mathrm{e}^{2x}\sin 4x \pm \int B\mathrm{e}^{2x}\sin 4x\,\mathrm{d}x$$

A1: Correct integration $\frac{1}{4}\mathrm{e}^{2x}\sin 4x - \int \frac{1}{2}\mathrm{e}^{2x}\sin 4x\,\mathrm{d}x$ which may be left unsimplified

dM1: Integrates again in the same direction to

$$\pm A\mathrm{e}^{2x}\sin 4x \pm B\mathrm{e}^{2x}\cos 4x \pm C\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x$$

A1: Fully correct $\frac{1}{4}\mathrm{e}^{2x}\sin 4x-\left(-\frac{1}{8}\mathrm{e}^{2x}\cos 4x+\int\frac{1}{4}\mathrm{e}^{2x}\cos 4x\,\mathrm{d}x\right)$ which may be left unsimplified

ddM1: Attempts to collect terms in $\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x$ following award of both previous M's

Condone arithmetical slips when combining the terms in $\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x$

A1: $\frac{1}{5}\mathrm{e}^{2x}\sin 4x+\frac{1}{10}\mathrm{e}^{2x}\cos 4x+c$ with or without the + c

Alternative scheme which can be applied in exactly the same way

Way Two: $\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x=\frac{1}{2}\mathrm{e}^{2x}\cos 4x+\int 2\mathrm{e}^{2x}\sin 4x\,\mathrm{d}x$	M1, A1
$=\frac{1}{2}\mathrm{e}^{2x}\cos 4x+\left(\mathrm{e}^{2x}\sin 4x-\int 4\mathrm{e}^{2x}\cos 4x\,\mathrm{d}x\right)$	dM1, A1
$5\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x=\mathrm{e}^{2x}\sin 4x+\frac{1}{2}\mathrm{e}^{2x}\cos 4x\Rightarrow\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x=\frac{1}{5}\mathrm{e}^{2x}\sin 4x+\frac{1}{10}\mathrm{e}^{2x}\cos 4x+c$	ddM1, A1

Note that attempts using the D and I method are acceptable. For example, under Way One, the first 4 marks are allocated as follows;

M2: Integrates to a form $\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x=\pm A\mathrm{e}^{2x}\sin 4x\pm B\mathrm{e}^{2x}\cos 4x\pm C\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x$

A2: Integrates correctly $\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x=\frac{1}{4}\mathrm{e}^{2x}\sin 4x+\frac{1}{8}\mathrm{e}^{2x}\cos 4x-\frac{1}{4}\int \mathrm{e}^{2x}\cos 4x\,\mathrm{d}x$

(ii)

M1: Fully applies the substitution WITH $k\mathrm{d}x=\cos u\,(\mathrm{d}u)$.

Look for $\int \pm k\frac{\sin^3 u}{\sqrt{1-\sin^2 u}}\times\cos u\,\mathrm{d}u$ You may condone a missing du only if the intention is clear

A1: $\int \frac{1}{8}\sin^3 u\,(\mathrm{d}u)$ which may be unsimplified but the $\frac{\cos u}{\sqrt{1-\sin^2 u}}$ must be fully cancelled.

Condone a missing du here and for all remainder of the question

dM1: Puts into a form that can be integrated. It is dependent upon having achieved

$\pm\int k\sin^3 u\ \mathrm{d}u$

$$\int \sin^3 u\ \mathrm{d}u = \int \sin u\left(1-\cos^2 u\right)\mathrm{d}u = \int \sin u - \sin u\cos^2 u\ \mathrm{d}u$$

You may see the substitution $\sin^3 u = \pm\frac{3}{4}\sin u \pm \frac{1}{4}\sin 3u$

ddM1: And then integrates to a correct form. Look for $\pm\alpha\cos u \pm \beta\cos^3 u$ or $\pm\alpha\cos u \pm \beta\cos 3u$

A1: $-\frac{1}{8}\cos u + \frac{1}{24}\cos^3 u$ or $\frac{3}{32}\cos u + \frac{1}{96}\cos 3u$

M1: States the correct limits or uses the correct limits (either way around) to an integrated form

If the limits are in degrees, this mark can only be awarded when the value of sin30, sin60, cos30 and cos60 are correctly applied to a function that must involve sin and/or cos only

A1: $\frac{3}{64}\sqrt{3} - \frac{11}{192}$ or $\frac{9\sqrt{3}-11}{192}$ but isw after a correct answer

Question Number	Scheme	Marks
8 (a) (i)	$18 = 140 - \dfrac{k}{0.3\times 0+8} \Rightarrow \dfrac{k}{8} = 122 \Rightarrow k = 976$	B1
(ii)	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = \dfrac{0.3\times 976}{\left(0.3\times 10+8\right)^2} = \text{awrt } 2.42 \left(\text{m}^2 / \text{day}\right)$	M1, A1
		(3)
(b)	$\displaystyle\int \frac{\mathrm{d}S}{128-S} = \int \frac{1}{50}\,\mathrm{d}t$	B1
	$-\ln(128-S) = \dfrac{1}{50}t + c$	M1, A1
	Uses $t = 0, S = 18 \Rightarrow -\ln 110 = c$	M1
	$\ln\left(\dfrac{110}{128-S}\right) = \dfrac{1}{50}t \Rightarrow \dfrac{128-S}{110} = \mathrm{e}^{-\frac{1}{50}t} \Rightarrow S = 128 - 110\mathrm{e}^{-\frac{1}{50}t}$	ddM1, A1
		(6)
(c)	128 (m^2)	B1
		(1)
		(10 marks)

(a)(i)

B1: Substitutes $t = 0$ $S = 18$ to reach $k = 976$

(a)(ii)

M1: Substitutes $t = 10$ into $\dfrac{\mathrm{d}S}{\mathrm{d}t} = \pm\dfrac{\delta}{\left(0.3t+8\right)^2}$ or if S is initially written as a single fraction, substitutes $t = 10$ into a $\dfrac{\mathrm{d}S}{\mathrm{d}t}$ of the form $\dfrac{\alpha\left(0.3t+8\right)-\beta\left(at+b\right)}{\left(0.3t+8\right)^2}$

A1: $\dfrac{1464}{605}$ or awrt 2.42 following M1. No requirement for units. If two answers are given then it is A0

(b)

B1: Correctly separates the variables. The dS and the dt must be correctly placed but there is no requirement for the integral signs

M1: Integrates to $P\ln(128-S) = Qt\left(+c\right)$ There is no need for the + c

A1: Correct integration including the + c

M1: Uses t =0 S =18 to find a non-zero value for c. There must have been some attempt to integrate