7. (i) Use integration by parts to find

$$\int e^{2x} \cos 4x \, dx$$

(6)

(ii) Using the substitution $2x = \sin u$ and algebraic integration, find the exact value of

$$\int_{\frac{1}{4}}^{\frac{\sqrt{3}}{4}} \frac{2x^3}{\sqrt{1-4x^2}} \, dx$$

giving the answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*)

(7)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 13 marks)

8. The growth of algae on the surface of a pond is being monitored.

The surface area of the pond covered by algae, $S\ \text{m}^2$, is modelled by the equation

$$S = 140 - \frac{k}{0.3t + 8}$$

where t days is the time after monitoring began and k is a constant.

Given that $18\ \text{m}^2$ of the surface area of the pond was covered by algae when monitoring began,

(a) find, according to this model,

(i) the value of k,

(ii) the value of $\frac{\text{d}S}{\text{d}t}$ when $t = 10$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

In an improved model, S increases with respect to t according to the differential equation

$$\frac{\text{d}S}{\text{d}t} = \frac{128 - S}{50}$$

(b) Solve the differential equation, with $S = 18$ when $t = 0$, to find an equation for S in terms of t.

(6)

Given that the algae grows until it covers the entire surface of the pond,

(c) find, according to the **improved** model, the maximum possible area of the pond.

(1)

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