Question Number	Scheme	Marks
5 (a)	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\-2\\-8\end{pmatrix}=2\times6+3\times-2+-2\times-8$	M1
	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\-2\\-8\end{pmatrix}=\begin{vmatrix}2\\3\\-2\end{vmatrix}\begin{vmatrix}6\\-2\\-8\end{vmatrix}\cos(BAC)\Rightarrow 22=\sqrt{17}\times\sqrt{104}\times\cos(BAC)\Rightarrow\cos(BAC)=\frac{22}{\sqrt{17}\sqrt{104}}$	dM1
	angle $BAC=58.5°$	A1
		(3)
(b)	Area triangle $ABC=\frac{1}{2}\times\sqrt{17}\times\sqrt{104}\sin58.5°=17.9$	M1, A1
		(2)
(c)	Attempts $\pm\overrightarrow{DC}=\begin{pmatrix}6-2\lambda\\-2-3\lambda\\-8+2\lambda\end{pmatrix}$ with any constant λ	M1, A1
	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix}\bullet\begin{pmatrix}6-2\lambda\\-2-3\lambda\\-8+2\lambda\end{pmatrix}=0\Rightarrow\lambda=\frac{22}{17}$	dM1
	$\overrightarrow{AD}=\frac{44}{17}\mathbf{i}+\frac{66}{17}\mathbf{j}-\frac{44}{17}\mathbf{k}$ o.e. $\frac{22}{17}\left(2\,\mathbf{i}+3\,\mathbf{j}-2\mathbf{k}\right)$	A1
		(4)
		(9 marks)

(a)

M1: Attempts the scalar product of $\pm\overrightarrow{AB}\bullet\pm\overrightarrow{AC}$.

Condone slips, e.g. in sign, but the intention must be clear.

If the calculation is not seen in full, expect to see 2 out of 3 of $\left(\pm12\right)+\left(\pm6\right)+\left(\pm16\right)$

If no intermediary calculation is seen it must be correct, so look for ±22

dM1: Full method to find $\cos(BAC)$ or BAC. It is dependent upon the previous M.

If the calculations for the modulus are not written out, at least one must be correct

A1: Angle $BAC=$ awrt $58.5°$. Note that $BAC=$ awrt $121.5°$ will be A0

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Alternatives exist, for example using the cosine rule

M1: Attempts the length of all 3 sides including an attempt at $\pm\overrightarrow{BC}$ and $\left|\pm\overrightarrow{BC}\right|$

$\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}=\left(6\mathbf{i}-2\mathbf{j}-8\mathbf{k}\right)-\left(2\mathbf{i}+3\mathbf{j}-2\mathbf{k}\right)=4\mathbf{i}-5\mathbf{j}-6\mathbf{k}$ leading to

$\left|\overrightarrow{BC}\right|=\sqrt{4^2+5^2+6^2}$

If no calculations are seen then score if 2 out of 3 are correct.

dM1: Full attempt at the cosine rule to find $\cos(BAC)$, BAC or $180-BAC$. E.g

$\cos(BAC)=\frac{17+104-77}{2\times\sqrt{17}\times\sqrt{104}}$

A1: Angle $BAC=$ awrt $58.5°$

(b)

M1: Attempts the area using a correct combination of lengths and angles. E.g.

$\frac{1}{2}\times\sqrt{17}\times\sqrt{104}\sin 58.5°$

Follow through on their lengths and angle from part (a)

A1: AWRT 17.9. Can be scored when angle $BAC=$ awrt $121.5°$

Alt (b)

You may see vector product used. $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&3&-2\\6&-2&-8\end{vmatrix}=-28\mathbf{i}+4\mathbf{j}-22\mathbf{k}$ then

$\frac{1}{2}\sqrt{28^2+4^2+22^2}=17.9$

M1: Full attempt condoning slips

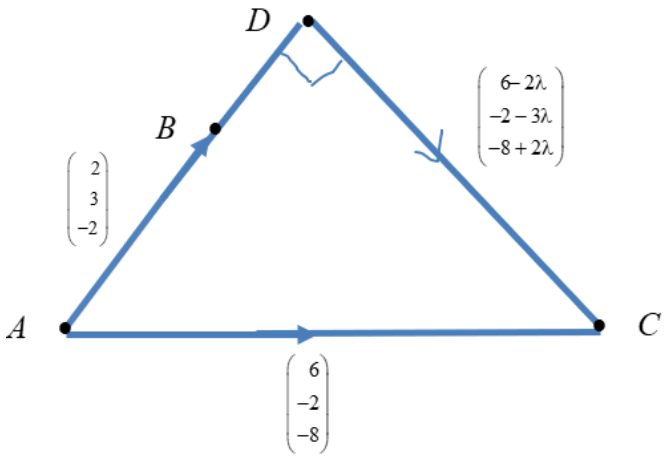
A1: $\sqrt{321}$ or AWRT 17.9

(c)

M1: Attempts $\pm\overrightarrow{DC}$ Look for a calculation that involves

- $\overrightarrow{AD}=\lambda\overrightarrow{AB}$ allowing with any constant
- use of $\overrightarrow{DC}=\overrightarrow{DA}+\overrightarrow{AC}$ or equivalent either way around

Condone slips but the intention must be clear



A1: Correct $\pm\overrightarrow{DC} = \pm\begin{pmatrix} 6-2\lambda \\ -2-3\lambda \\ -8+2\lambda \end{pmatrix}$

dM1: Uses a correct attempt at the scalar product of $\pm\overrightarrow{AD} \bullet \pm\overrightarrow{DC}$, sets equal to 0 and reaches a value for their constant 'λ'. Dependent upon previous M

A1: Correct $\overrightarrow{AD}$ using correct notation. Allow either version but not a mixture or coordinates

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Alternative methods exist and can be scored in a similar way with the first three marks for a correct method of finding the **exact** value for $\lambda\left(=\frac{22}{17}\right)$.

For example, using the trigonometry of a right-angled triangle (and possibly Pythagoras' theorem too).

M1: For attempting the length of AD using $\left|\overrightarrow{AC}\right|\cos BAC = \sqrt{104}\cos BAC$ allowing for use of degrees here

A value must be found, exact or approximate. An alternative is to use

$\left|\overrightarrow{AC}\right|\sin\left(90-BAC\right) = '\sqrt{104}'\sin'31.5'$

A1: Correct and exact length $\left|\overrightarrow{AD}\right| = \left|\overrightarrow{AC}\right| \times \cos BAC = \sqrt{104} \times \frac{22}{\sqrt{17}\sqrt{104}} = \frac{22}{\sqrt{17}}$

dM1: Uses ratio $\overrightarrow{AD} : \overrightarrow{AB} = \frac{22}{\sqrt{17}} : \sqrt{17} = \frac{22}{17} : 1$ to find the exact value of λ.

Other alternatives exist such as $\left(2\lambda\right)^2 + \left(3\lambda\right)^2 + \left(-2\lambda\right)^2 = \left(\frac{22}{\sqrt{17}}\right)^2 \Rightarrow \lambda = ...$

A1: Hence $\overrightarrow{AD} = \frac{22}{17} \times \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{44}{17} \\ \frac{66}{17} \\ -\frac{44}{17} \end{pmatrix}$ allow alternatives such as $\overrightarrow{AD} = \frac{22}{17} \times \left(2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\right)$

Question Number	Scheme	Marks
6 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}(12y) = 12\frac{\mathrm{d}y}{\mathrm{d}x}$	B1
	$4y^2 + 8xy\frac{\mathrm{d}y}{\mathrm{d}x} + 12\frac{\mathrm{d}y}{\mathrm{d}x} + 6 = 0$	M1, A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4y^2 - 6}{8xy + 12} = \frac{-2y^2 - 3}{4xy + 6}$	M1, A1
		(5)
(b)	Deduces that $4xy + 6 = 0$	B1ft
	Substitutes $y = -\frac{3}{2x}$ into $4xy^2 + 12y + 6x = 53$	M1
	$\Rightarrow \frac{9}{x} - \frac{18}{x} + 6x = 53 \Rightarrow 6x^2 - 53x - 9 = 0$	A1
	$\Rightarrow (6x+1)(x-9) = 0 \Rightarrow x = ...$	dM1
	$\Rightarrow a = 9, b = -\frac{1}{6}$ but allow $P = \left(9, -\frac{1}{6}\right)$	A1
		(5)
		(10 marks)

(a)

B1: Correct differentiation $\frac{\mathrm{d}}{\mathrm{d}x}(12y) = 12\frac{\mathrm{d}y}{\mathrm{d}x}$ but allow equivalents such as $12y'$

M1: Correct attempt at the product rule on $4xy^2 \rightarrow pxy\frac{\mathrm{d}y}{\mathrm{d}x} + qy^2$

A1: Correct differentiation.

If they write $\frac{\mathrm{d}y}{\mathrm{d}x} = 4y^2 + 8xy\frac{\mathrm{d}y}{\mathrm{d}x} + 12\frac{\mathrm{d}y}{\mathrm{d}x} + 6$ it is A0 unless further work suggests that this is not what they meant. Allow y' or $\dot{y}$ for $\frac{\mathrm{d}y}{\mathrm{d}x}$. You may also see $4y^2\mathrm{d}x + 8xy\,\mathrm{d}y + 12\,\mathrm{d}y + 6\mathrm{d}x = 0$ which is fine.

M1: Collects exactly two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms together, one from each from the differentiation of $4xy^2$ and $12y$, and makes $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject. Note that the differentiation of the product rule could be incorrect e.g. $4xy^2 \rightarrow 8xy\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2y^2 - 3}{4xy + 6}$ o.e such as $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2y^2 + 3}{4xy + 6}$ but must be in simplest form