Question Number	Scheme	Marks
7 (a)	$(1, 4.5)$	B1B1 **(2)**
(b)	Attempts $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{12\sin\theta\cos\theta}{-24\cos^2\theta\sin\theta} = \left(-\dfrac{1}{2\cos\theta}\right)$	M1A1
	Subs $\theta = \dfrac{\pi}{3}$ into $\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = ...(=-1)$	dM1
	Uses gradient of normal with $(1, 4.5) \Rightarrow (y-4.5) = 1(x-1)$	ddM1
	$y = x + 3.5$	A1 **(5)**
(c)	Attempts $\int y\dfrac{dx}{d\theta}\,d\theta = \int 6\sin^2\theta \times -24\cos^2\theta\sin\theta\,d\theta$	M1A1
	Uses $\sin^2\theta = 1-\cos^2\theta \Rightarrow \int y\dfrac{dx}{d\theta}\,d\theta = \int A\left(\cos^2\theta\sin\theta - \cos^4\theta\sin\theta\right)d\theta$	dM1
	Area of trapezium= $\dfrac{1}{2}(3.5+4.5) = 4$	B1ft
	Attempts trapezium + area under curve $= \dfrac{1}{2}(3.5+4.5) - 144\int_{\frac{\pi}{3}}^{0} \sin^3\theta\cos^2\theta\,d\theta$	dM1
	Area $= 4 + 144\int_0^{\frac{\pi}{3}} \left(\sin\theta\cos^2\theta - \sin\theta\cos^4\theta\right)d\theta$ *	A1* **(6)**
(d)	Area $= 4 + 144\left[-\dfrac{\cos^3\theta}{3} + \dfrac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{3}} = 4 + 144\left(\left(-\dfrac{1}{24} + \dfrac{1}{160}\right) - \left(-\dfrac{1}{3} + \dfrac{1}{5}\right)\right) = \dfrac{181}{10}$	M1A1A1 **(3)**
		(16 marks)

(a)

B1: Either of $(1, 4.5)$ Accept any exact equivalent for 4.5 (May be seen on a diagram) May be written separately e.g. $x = 1,\ y = 4.5$ Condone coordinates without brackets

B1: Both $(1, 4.5)$ Accept any exact equivalent for 4.5 (May be seen on a diagram) May be written separately e.g. $x = 1,\ y = 4.5$ Condone coordinates without brackets. isw once a correct pair is seen

Note for parts (b), (c) and (d) do not penalise incorrect notation provided the intention is clear or notation is recovered. E.g. $\sin\theta^2$ **for** $\sin^2\theta$**.**

(b)

M1: Attempts $\left(\dfrac{dy}{dx} =\right) \dfrac{dy/d\theta}{dx/d\theta}$ Allow e.g. $"12\sin\theta\cos\theta" \times \dfrac{1}{-24\cos^2\theta\sin\theta}$ or e.g. $"6\sin 2\theta" \times \dfrac{1}{-12\sin 2\theta\cos\theta}$ Condone poor differentiation on y and/or x provided the functions are both "changed". If the attempts at $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$ are not seen separately then look for the formula stated or a

numerator of the form $p\sin\theta\cos\theta$ or a denominator of the form $q\cos^2\theta\sin\theta$. May be seen as an attempt to directly find $-\frac{\mathrm{d}x}{\mathrm{d}y}$

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{12\sin\theta\cos\theta}{24\cos^2\theta\sin\theta}=\left(-\frac{1}{2\cos\theta}\right)$ oe May be unsimplified and may be implied by further work

dM1: Substitutes $\theta=\frac{\pi}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to achieve a value. If just a value is stated then you may need to check this on your calculator. Condone slips provided the intention is clear. Dep. on 1st M. May be implied by their gradient in the equation of a normal.

ddM1: Attempts to find the normal by using the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their $(1,4.5)$ to produce an equation of a normal $\Rightarrow (y-4.5)=1(x-1)$ If using $y=mx+c$ then $\Rightarrow c=...$ Dep on both previous Ms

A1: $y=x+3.5$ o.e. in the required form

Alt(b) Finding the cartesian equation and differentiating

M1: Finds the cartesian equation $y=A\pm Bx^{\frac{2}{3}}$ and attempts to differentiate proceeding to the form $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)Cx^{-\frac{1}{3}}$ May be unsimplified and index does not need to be processed

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-x^{-\frac{1}{3}}$ May be unsimplified but index processed

dM1: Substitutes $x=1$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}=(-1)$ to achieve a value. If just a value is stated then you may need to check this on your calculator. Condone slips provided the intention is clear. Dep. on 1st M. May be implied by their gradient in the equation of a normal.

ddM1: Attempts to find the normal by using the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their $(1,4.5)$ to produce an equation of a normal $\Rightarrow (y-4.5)=1(x-1)$ If using $y=mx+c$ then $\Rightarrow c=...$ Dep on both previous Ms

A1: $y=x+3.5$ o.e. in the required form

(c) Note: Do not be concerned with the order of the limits on the integral and being consistent with 144 until the final mark.

M1: Attempts $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$ (Allow omission of $\mathrm{d}\theta$ and allow unsimplified). Condone poor attempts at $\frac{\mathrm{d}x}{\mathrm{d}\theta}$

A1: $\int 6\sin^2\theta\times -24\cos^2\theta\sin\theta\,(\mathrm{d}\theta)$

dM1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ to proceed from $\int A\sin^3\theta\cos^2\theta\,\mathrm{d}\theta$ to produce an expression in an 'integrable form'. Dep. on 1st M

$\int A\sin^3\theta\cos^2\theta\,\mathrm{d}\theta = \int A\sin\theta\left(1-\cos^2\theta\right)\cos^2\theta\,\mathrm{d}\theta = A\int\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)\mathrm{d}\theta$ Condone sign slips only for this mark.

B1ft: Area of trapezium$=\frac{1}{2}("3.5"+"4.5")$ or alternatively $\int_0^1("x+3.5")\mathrm{d}x=\left[\frac{x^2}{2}+3.5x\right]_0^1\left(=0.5+3.5\right)=4$

Where integration is used you must see the integrated expression but they can proceed directly to the answer from this point. You may need to check this on your calculator.

Must see any correct calculation to find the area. Just stating area of trapezium = "4" is B0ft

dM1: Attempts trapezium + area under curve. Look for $="\frac{1}{2}(3.5+4.5)"\pm A\int_0^{\frac{\pi}{3}}\sin^3\theta\cos^2\theta\left(\mathrm{d}\theta\right)$

OR alternatively $="\frac{1}{2}(3.5+4.5)"\pm A\int_0^{\frac{\pi}{3}}\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)\left(\mathrm{d}\theta\right)$ There must be an attempt to use the correct limits either way around and add a trapezium. Dep on 1st M

A1*: Achieves the given answer including $\mathrm{d}\theta$ provided all previous marks have been awarded. **Must be seen in (c) so do not condone at the start of (d).** Withhold this mark if the limits written on the integral are not consistent with ± 144 oe on any line of their solution but do not be concerned with the positions of 8 and 1 on the integral before they are replaced. Do not condone sign slips but allow invisible brackets to be recovered before the final answer.

Alt(c) – finding the cartesian equation first

M1: Finds the cartesian equation and proceeds to the form $\int A\pm Bx^{\frac{2}{3}}\,\mathrm{d}x$ oe (ignore limits)

A1: $\int 6-\frac{3}{2}x^{\frac{2}{3}}\,(\mathrm{d}x)\left(=\frac{3}{2}\int 4-x^{\frac{2}{3}}\,\mathrm{d}x\right)$ oe

dM1: $\frac{3}{2}\int\left(4-\left(8\cos^3\theta\right)^{\frac{2}{3}}\right)(-24\cos^2\theta\sin\theta)\Rightarrow$ integrable form as in main scheme notes

B1ftdM1A1*: Main scheme notes

(d)

M1: $\int\sin\theta\cos^n\theta=\pm\frac{\cos^{n+1}\theta}{n+1}$ in either term. This cannot be implied by a correct answer. May use substitution e.g. $u=\cos\theta$

A1: Any correct (unsimplified) answer following a correct integrated expression. Trigonometric expressions should be evaluated.

A1: 18.1 or exact equivalent cso

Question Number	Scheme	Marks
8 (a)	$4p^2 - q^2 = (2p+q)(2p-q)$	B1
		(1)
(b)	Makes one valid statement :Either $\begin{matrix}2p+q=46\\2p-q=1\end{matrix}$ **or** $\begin{matrix}2p+q=23\\2p-q=2\end{matrix}$	M1
	Solves correctly one of the pairs (see notes): e.g. $2p+q=46,\ 2p-q=1 \Rightarrow\ p=11.75, (q=22.5)$ oe **or** e.g. $2p+q=23,\ 2p-q=2 \Rightarrow p=6.25, (q=10.5)$ oe	A1
	States and attempts to solve **both equations (see above)**	dM1
	Fully correct proof by contradiction requiring (1) correct set up, (2) correct solution of both equations and (3) correct conclusion *	A1*
		(4)
		Total 5

(a)

B1: Correct factorisation

Note on epen this is B1M1A1A1* but we are marking this M1A1dM1A1*

Mark the attempts at valid pairs of equations in the order which scores the most marks

(b) If you see any other proofs which appear to be credit worthy then send to review

M1: Deduces either of the valid pairs of equations. There is no requirement to justify that $2p+q$ must be greater than $2p-q$ and therefore solve the other possible equations. Ignore these attempts at these pairs of equations.

A1: Correctly solves one of the two valid pairs of equations. No working is required, just the correct solution. Note that it is acceptable to only go as far as finding either one of p or q as that is a non-integer. However, if they find the other one of p or q then they must both be correct. Beware that the correct solving may be their second valid pair.

Note if they only proceed to e.g. $4p=25$ **they must explain the contradiction** that e.g. p cannot be an integer **because** 25 is not divisible by 4 or LHS=even, RHS=odd or LHS=multiple of 4, RHS=not a multiple of 4

dM1: States and attempts to solve **both** valid pairs of equations. Condone proceeding as far as e.g. $4p=47$ for this mark. It is dependent on the previous method mark.

A1*: Full proof by contradiction. This must follow M1A1dM1

Look for

- a correct set up with words such as "assume there is" or "let there be" with "integers p and q"
- correct solution of **both** valid pairs of equations (and explanations where required e.g. if only proceeding as far as $4p=25$) If p and q are both found for both valid pairs then they must be correct.
- a (minimal) conclusion