7.

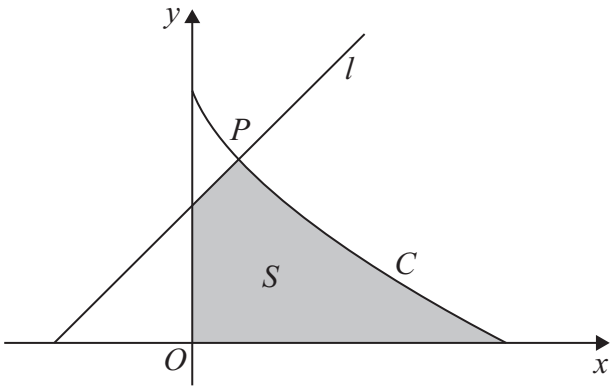

Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta \qquad y = 6\sin^2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Find an equation for l in the form $y = mx + c$, where m and c are constants.

(5)

The finite region S, shown shaded in Figure 1, is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(6)

(d) Hence, using algebraic integration, find the exact area of S.

(3)

(Total for Question 7 is 16 marks)

8. (a) Factorise $4p^2 - q^2$

(1)

(b) Hence, prove by contradiction, that there are no positive integers p and q such that

$$4p^2 - q^2 = 46$$

(4)

(Total for Question 8 is 5 marks)

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9. $ABCD$ is a parallelogram with AB parallel to DC and AD parallel to BC.
The position vectors of A, B, C and D relative to a fixed origin O are
a, **b**, **c** and **d** respectively.

Given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k} \qquad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + 6\mathbf{k} \qquad \mathbf{c} = -\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$$

(a) find the position vector **d**,

(3)

(b) find the angle between the sides AB and BC of the parallelogram,

(4)

(c) find the area of the parallelogram $ABCD$.

(2)

The point E lies on the line through the points C and D, so that D is the midpoint of CE.

(d) Use your answer to part (c) to find the area of the trapezium $ABCE$.

(2)

(Total for Question 9 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS