5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 10 \\ c \\ 3 \end{pmatrix} + \mu \begin{pmatrix} a \\ b \\ -2 \end{pmatrix}$$

where a, b and c are constants and λ and μ are scalar parameters.

Given that

- l_1 and l_2 meet when $\lambda = -2$
- l_1 and l_2 are perpendicular

find the value of a, the value of b and the value of c.

(6)

(Total for Question 5 is 6 marks)

---

6. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Use algebraic integration and the substitution $u = 2x - 1$ to show that

$$\int_2^5 \frac{(3x+2)^2}{2x-1}\,\mathrm{d}x = 72 + \frac{49}{8}\ln 3$$

(6)

The curve C has equation

$$y = \frac{3x+2}{2\sqrt{2x-1}} \qquad x > 1$$

The finite region R is bounded by C, the x-axis and the lines with equations $x = 2$ and $x = 5$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using the result from part (a), find the exact value of the volume of the solid generated.

(2)

(Total for Question 6 is 8 marks)

7.

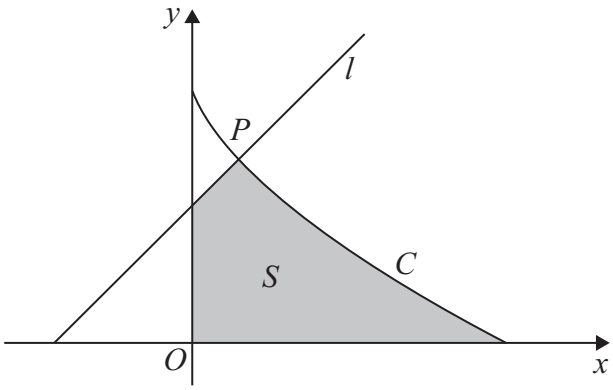

Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta \qquad y = 6\sin^2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Find an equation for l in the form $y = mx + c$, where m and c are constants.

(5)

The finite region S, shown shaded in Figure 1, is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(6)

(d) Hence, using algebraic integration, find the exact area of S.

(3)

(Total for Question 7 is 16 marks)