3. The height, h metres, of a shrub, t years after it was planted, is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{2h^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $h = 1$ when $t = 1$

(a) solve the differential equation to show that

$$h = \frac{at^2}{(1 + bt)^2}$$

where a and b are constants to be found.

(6)

(b) Hence find, according to the model, the limit of the height of the shrub.

(2)

(Total for Question 3 is 8 marks)

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4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Find

$$\int_1^3 \frac{3x^2 + 8}{x^2 - 4x}\,\mathrm{d}x$$

giving the answer in the form $a + b\ln 3$ where a and b are constants.

(7)

(Total for Question 4 is 7 marks)

5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 10 \\ c \\ 3 \end{pmatrix} + \mu \begin{pmatrix} a \\ b \\ -2 \end{pmatrix}$$

where a, b and c are constants and λ and μ are scalar parameters.

Given that

- l_1 and l_2 meet when $\lambda = -2$
- l_1 and l_2 are perpendicular

find the value of a, the value of b and the value of c.

(6)

(Total for Question 5 is 6 marks)

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6. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Use algebraic integration and the substitution $u = 2x - 1$ to show that

$$\int_2^5 \frac{(3x+2)^2}{2x-1}\,\mathrm{d}x = 72 + \frac{49}{8}\ln 3$$

(6)

The curve C has equation

$$y = \frac{3x+2}{2\sqrt{2x-1}} \qquad x > 1$$

The finite region R is bounded by C, the x-axis and the lines with equations $x = 2$ and $x = 5$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using the result from part (a), find the exact value of the volume of the solid generated.

(2)

(Total for Question 6 is 8 marks)
