M1: Uses correct algebra starting from their integrated expression of the form $\alpha h^{-\frac{1}{2}} = \beta t^{-1} + c$ oe to make $h$ the subject . Look for an attempt that has

- a single fraction being formed before "inverting" oe. Do not allow attempts where the candidate inverts each term. Alternatively they may multiply throughout and factorise. Condone sign slips only in their manipulation
- a correct attempt to square i.e. each side is squared rather than each term. Condone the slip e.g. $5t^2$ instead of $25t^2$

A1: cao $h = \dfrac{25t^2}{\left(1+4t\right)^2}$ $\left(a = 25,\ b = 4\right)$

**(b)**

M1: Max $h = \dfrac{\text{Their } a}{\text{Their } b^2}$ from an answer to (a) in the form $h = \dfrac{at^2}{(c+bt)^2}$ $a > 0$ or allow from an equation of the required form stated in this part.

Implied by 1.56 (i.e. answer to 3sf)

A1ft: $\dfrac{25}{16}$ (m) or exact equivalent e.g. or 1.5625 (m) following through on their $h = \dfrac{"25"t^2}{\left(1+"4"t\right)^2}$ oe $a > 0$

Condone e.g. $h \leqslant \dfrac{25}{16}$ or $0 < h < \dfrac{25}{16}$ or any other correct notation including interval notation

e.g. $\left(0, \dfrac{25}{16}\right)$, $\left(0, \dfrac{25}{16}\right]$ Units not required but if given they must be correct.

| Question number | Scheme | Marks |
|---|---|---|
| **4** | For fractions $\frac{A}{x}+\frac{B}{x-4}=-\frac{2}{x}+\frac{14}{x-4}$ | M1 A1 |
| | $\frac{3x^2+8}{x^2-4x}=3-\frac{2}{x}+\frac{14}{x-4}$ | B1ft |
| | $\int_1^3 \frac{3x^2+8}{x^2-4x}\,\mathrm{d}x=\int_1^3 3-\frac{2}{x}+\frac{14}{x-4}\,\mathrm{d}x=\left[3x-2\ln\lvert x\rvert+14\ln\lvert x-4\rvert\right]_{(x=1)}^{(x=3)}$ | M1 A1ft |
| | $=9-2\ln 3-3-14\ln 3$ | dM1 |
| | $=6-16\ln 3$ | A1 |
| | | **(7)** |
| | | **[7 marks]** |

M1: Attempts to write in partial fractions form and finds values $A$ **and** $B$ for $\frac{A}{x}+\frac{B}{x-4}$ $A,B\neq 0$ Note this may be scored without the "3". They cannot just state values for $A$ and $B$ for this mark – it must be written in partial fraction form.

A1: $-\frac{2}{x}+\frac{14}{x-4}$ oe

B1ft: Correct partial fraction form $\frac{3x^2+8}{x^2-4x}=3-\frac{2}{x}+\frac{14}{x-4}$ oe such as $3-\frac{2}{x}-\frac{14}{4-x}$ **ft on their non-zero values for *A* and *B***. Condone e.g. $\frac{3x^2+8}{x^2-4x}=3+-\frac{2}{x}+\frac{14}{x-4}$

Allow use of the letters "*A*" and "*B*" instead of numerical values i.e. $3+\frac{A}{x}+\frac{B}{4-x}$

Also allow this mark to be scored for $3+\frac{12x+8}{x^2-4x}$ or ft on their remainder $Cx+D$ both non-zero.

M1: For $\int\frac{1}{x}\,\mathrm{d}x\to\ln x$ **and** $\int\frac{1}{x-4}\,\mathrm{d}x\to\ln\lvert x-4\rvert$ but allowing $\ln(x-4)$ and $-\ln(4-x)$. Condone invisible brackets for this mark.

A1ft For $\int 3-\frac{2}{x}+\frac{14}{x-4}\,\mathrm{d}x\to 3x-2\ln\lvert x\rvert+14\ln\lvert x-4\rvert$ oe following through on their non-zero coefficients. $x-4$ requires modulus signs and/or brackets unless implied by later work (condone not around $x$)

E.g. allow $\int 3-\frac{2}{x}-\frac{14}{4-x}\,\mathrm{d}x\to 3x-2\ln x+14\ln(4-x)$

dM1: Uses both limits 3 and 1, subtracts either way round and proceeds to the expression $P+Q\ln 3$ where $P$ and $Q$ are rational and non-zero. Must have simplified to this form.

It cannot be implied by $6-16\ln 3$ on its own. We must see some evidence of limits being used which as a minimum is the limits substituted in the integrand.

It is dependent on the previous method mark.

A1: $6-16\ln 3$ **following all previous marks scored**

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**Special Case:**

Some candidates know to use PF but fail to see it is an improper fraction.

Note that use of $3x^2+8\equiv A(x-4)+Bx$ would result in $A=-2, B=14$ via substitution. The solution may appear as:

$$\frac{3x^2+8}{x^2-4x}=\frac{14}{x-4}-\frac{2}{x}$$

$$\int_1^3 \frac{3x^2+8}{x^2-4x}\,\mathrm{d}x=\int_1^3 \frac{14}{x-4}-\frac{2}{x}\,\mathrm{d}x=\Big[14\ln|x-4|-2\ln|x|\Big]_{(x=1)}^{(x=3)}$$

$$=14\ln 1-2\ln 3-(14\ln 3-2\ln 1)=-16\ln 3$$

This potentially will score M1A1B0M1A0dM0A0 for 3 out of 7

**However, a candidate who correctly adds 3 to their partial fraction can potentially score full marks using the main scheme – send to review if unsure.**

| Question Number | Scheme | Marks |
|---|---|---|
| **5** | Point of intersection is $(6,-1,-5)$ or the value of $\mu = 4$ | B1 |
| | Uses their $\mu$ and **i** components $10+\mu a = "6"$ to find $a$ | M1 |
| | $a = -1$ | A1 |
| | Uses perpendicular vectors $\Rightarrow \begin{pmatrix}-2\\1\\4\end{pmatrix}.\begin{pmatrix}a\\b\\-2\end{pmatrix} = 0 \Rightarrow -2a+b-8=0 \Rightarrow b=(6)$ | dM1 |
| | Uses **j** component $"c+\mu b" = "-1" \Rightarrow c+24=-1 \Rightarrow c=(-25)$ | ddM1 |
| | $b=6, c=-25$ | A1 |
| | | **(6)** |
| | | **[6 marks]** |

B1: States or uses the point of intersection as $(6,-1,-5)$ or the value of $\mu = 4$ Allow values or vectors seen or used or implied. These values are sometimes embedded within the work e.g. $2-2(-2)$, $1-2$ and $3+4(-2)$

M1: A full method to find "$a$". This involves

- using the **k** component of $l_2$ to find $\mu$
- followed by the **i** component to find $a$

A1: $a = -1$ May be seen or implied by further work.

dM1: A full method of finding "$b$".

This involves using the fact that $\begin{pmatrix}-2\\1\\4\end{pmatrix}.\begin{pmatrix}a\\b\\-2\end{pmatrix} = 0$ and using their value of $a$. Look for at least 2 correct products or implied by a correct value for $b$.

Condone arithmetical slips. It is dependent on the previous method mark.

ddM1:A full method of finding "$c$" using the **j** component. For using their values for $a$, $b$ and $\mu$ and proceeding from $"c+\mu b" = "-1"$ to a value for $c$. If just a value for $c$ is stated you may have to check using their values for $a$, $b$ and $\mu$. It is dependent on the previous two method marks.

A1: $b = 6, c = -25$

**(Note a common misread is $\lambda = 2$ and will lead to a point of intersection $(-2, 3, 11)$ of values of $\mu = -4,\ a = 3,\ b = 14,\ c = 59$ apply the misread rule in situations like this so max score will be B0M1A0dM1ddM1A1 – send to review if unsure)**

**If they attempt to find *c* before *b* then send to review (unless fully correct and scores full marks)**