1.

$$f(x) = \frac{27}{(3-5x)^2} \qquad |x| < \frac{3}{5}$$

(a) Find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3
Give each coefficient in its simplest form.

(4)

Using your answer to part (a), find the series expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \frac{27}{(3+5x)^2} \qquad |x| < \frac{3}{5}$

(1)

(c) $h(x) = \frac{27}{(3-x)^2} \qquad |x| < 3$

(2)

(Total for Question 1 is 7 marks)

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2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve has equation

$$4x^2 - y^2 + 2xy + 5 = 0$$

Given that

- points P and Q lie on the curve
- $\frac{dy}{dx} = 2$ at P and at Q

find the coordinates of P and Q.

(7)

(Total for Question 2 is 7 marks)

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3. The height, h metres, of a shrub, t years after it was planted, is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{2h^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $h = 1$ when $t = 1$

(a) solve the differential equation to show that

$$h = \frac{at^2}{(1 + bt)^2}$$

where a and b are constants to be found.

(6)

(b) Hence find, according to the model, the limit of the height of the shrub.

(2)

(Total for Question 3 is 8 marks)

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4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Find

$$\int_1^3 \frac{3x^2 + 8}{x^2 - 4x}\,\mathrm{d}x$$

giving the answer in the form $a + b\ln 3$ where a and b are constants.

(7)

(Total for Question 4 is 7 marks)