Question Number	Scheme	Marks
8 (a)	States/ uses $V=\lambda r^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$ OR $\dfrac{\mathrm{d}V}{\mathrm{d}t}=-C$	M1
	Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}r}\times\dfrac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow -C=\mu r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{C}{4\pi r^2}=-\dfrac{k}{r^2}$	A1*
		(3)
(b)	$\dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{k}{r^2} \Rightarrow \int r^2\mathrm{d}r=\int -k\ \mathrm{d}t$	
	$\dfrac{1}{3}r^3=-kt+c$	M1 A1
	Uses $t=0,\ r=30$ $\quad 9000=c$	M1
	Substitutes $r=12, t=24$ $\quad \dfrac{1}{3}\times 12^3=-k\times 24+9000 \Rightarrow k=(351)$	ddM1
	$r=\sqrt[3]{27000-1053t}$	A1
		(5)
(c)	$27000-1053T=0 \Rightarrow T=\ldots$	M1
	awrt 25.6	A1
		(2)
		(10 marks)

(a)

M1: States or uses either $\dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$ with any constant value for μ or any constant e.g. μ (allow any letter)

or $\dfrac{\mathrm{d}V}{\mathrm{d}t}=-C$ (Allow with k) but **must be a negative** and not given a value such as 1

If the formula for V is used (attempted) condone slips but it must obey $V=\lambda r^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$

dM1: Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}r}\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$ o.e. with $\dfrac{\mathrm{d}V}{\mathrm{d}t}=-C$ AND $\dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$

$\Rightarrow -C=\mu r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$

See the first M1 for conditions on $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}V}{\mathrm{d}r}$

A1*: Proceeds to $\dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{k}{r^2}$ via a correct method. Look for $-C=4\pi r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow \dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{C}{4\pi r^2}=-\dfrac{k}{r^2}$

Condone a proof such as $-C=\mu r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow \dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{C}{\mu r^2}=-\dfrac{k}{r^2}$

If candidates had originally used k they must explain why $-\dfrac{k}{4\pi r^2}$ can be replaced by $-\dfrac{k}{r^2}$

If the formula for V is used it must be correct $V=\dfrac{4}{3}\pi r^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$

(b)

Case 1: If candidates use any value for k e.g 0.75 (apart from the correct one) they can only score the first two marks in (b). Candidates using $k = 351$ can score all marks.

Case 2: Some candidates have an extra 4π and start (b) with $\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{4\pi r^2} \Rightarrow \int 4\pi r^2 \mathrm{d}r = \int -k \ \mathrm{d}t$

This should not affect the solution, so condone with any multiple of a constant for all marks

M1: Separates the variables (condoning lack of integral sign) and integrates to a form $...r^3 = ...t(+c)$

Condone the lack of the constant of integration.

A1: Achieves $\frac{1}{3}r^3 = -kt + c$ or equivalent including the $+ c$

Note that $r^3 = -3kt + c$ is also correct

M1: Substitutes $t = 0$ and $r = 30$ in their $r^3 = -3kt + c$ to find a non zero value for c.

This may be awarded following any attempt to integrate.

So $t = 0,\ r = 30$ in their $\Rightarrow \mathrm{f}(r) = \mathrm{g}(t) + c \Rightarrow c = ...$.

ddM1: Substitutes $r = 12$ and $t = 24$ with their non zero value of c to find a non zero value for k.

It is dependent upon BOTH previous M marks. Condone transcription errors on the value of c

A1: $r = \sqrt[3]{27000 - 1053t}$

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You may see k and c found in a different order.

For example, $\int_{30}^{12} r^2 \mathrm{d}r = \int_{0}^{24} -k \ \mathrm{d}t \Rightarrow \frac{1}{3}\left(12^3 - 30^3\right) = -k\left(24 - 0\right) \Rightarrow k = 351$

In cases like this the 2nd M mark would be awarded for the calculation of k

Hence equation would be $\frac{1}{3}r^3 = -351t + c \Rightarrow r^3 = -1053t + c$

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(c)

M1: Sets $r = 0$ and finds a value for T.

The equation must be of the form $r = \sqrt[n]{A - Bt}$ o.e. with $A,\ B > 0$ and n a positive integer

A1: Awrt 25.6 .

Alt (c) Using the fact that the volume decreases at a constant rate

M1: Finds the rate of decrease in volume $\frac{36000\pi - 2304\pi}{24} = (1404\pi)$ and uses this to find the time $\frac{36000\pi}{1404\pi}$

A1: Awrt 25.6

Question Number	Scheme	Marks
9 (a)	$k = \dfrac{\pi}{2}$	B1
	Attempts $y^2 \dfrac{\mathrm{d}x}{\mathrm{d}t} = (5\sin 2t)^2 \, 6\cos t = 150\sin^2 2t\cos t$	M1
	$= 600\sin^2 t\cos^3 t$	dM1
	$V = 600\pi \displaystyle\int_0^{\frac{\pi}{2}} \sin^2 t\cos^3 t \, \mathrm{d}t$	A1
		(4)
(b)	$\displaystyle\int \sin^2 t\cos^3 t \, \mathrm{d}t = \int \sin^2 t\left(1-\sin^2 t\right)\cos t \, \mathrm{d}t$	
	$= \displaystyle\int \sin^2 t\cos t - \sin^4 t\cos t \, \mathrm{d}t$	M1
	$= \dfrac{1}{3}\sin^3 t - \dfrac{1}{5}\sin^5 t$	dM1
	$V = 600\pi\left[\dfrac{1}{3}\sin^3 t - \dfrac{1}{5}\sin^5 t\right]_0^{\frac{\pi}{2}} = 600\pi\left(\dfrac{1}{3}-\dfrac{1}{5}\right) = 80\pi$	ddM1, A1
		(4)
		(8 marks)

(a)

B1: States or uses $k = \dfrac{\pi}{2}$. This may be scored from work in part (b) if not seen in (a)

M1: Attempts $y^2 \dfrac{\mathrm{d}x}{\mathrm{d}t} = (5\sin 2t)^2 \ldots\cos t = \ldots\sin^2 2t\cos t$

Condone missing brackets

dM1: Attempts to use $\sin 2t = 2\sin t\cos t$ in an expression of the form $\ldots\sin^2 2t\cos t$ and reaches $y^2 \dfrac{\mathrm{d}x}{\mathrm{d}t} = \ldots\sin^2 t\cos^3 t$. Condone $\sin^2 2t \to 2\sin^2 t\cos^2 t$ here

A1: $V = 600\pi \displaystyle\int_0^{\frac{\pi}{2}} \sin^2 t\cos^3 t \, \mathrm{d}t$ but allow this to be awarded with any k or in fact a constant k

The dt must be present on the final line