8. A large spherical balloon starts to deflate.

The balloon remains spherical whilst deflating.

At time t seconds after the balloon starts to deflate

- the balloon has radius r cm
- the balloon has volume V cm^3

Given that the volume of the balloon is modelled to decrease at a constant rate,

(a) use the model to show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 30 cm
- after 24 seconds the radius of the balloon is 12 cm

(b) solve the differential equation in part (a) to find an equation for r in terms of t.

(5)

Given that $V = 0$ when $t = T$

(c) use the model to find the value of T to one decimal place.

(2)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

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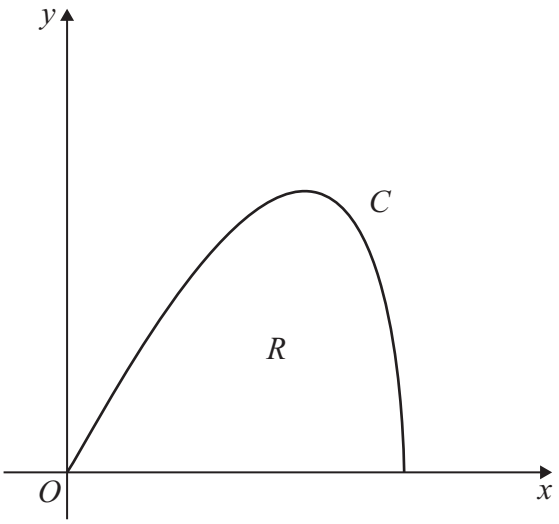

Figure 3

The curve C, shown in Figure 3, has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant k$$

where k is a constant.

The region R, also shown in Figure 3, is bounded by C and the x-axis.

This region is rotated through 2π radians about the x-axis to form a solid.

(a) Show that the volume, V, of this solid is given by

$$V = P\int_0^k \sin^m t \cos^n t \, \mathrm{d}t$$

where k, P, m and n are constants to be found.

(4)

(b) Hence, using algebraic integration, find the exact value of V.

(4)

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