Question Number	Scheme	Marks
7(a)	$x^2 \tan\frac{\pi}{4} + \frac{32}{\pi^2}\times\left(\frac{\pi}{4}\right)^2 = 11 \Rightarrow x^2 = (9)$	M1
	$x = \pm 3$	A1
		(2)
(b)	Attempt at chain rule $\quad y^2 \to ...y\frac{dy}{dx}$	M1
	Attempt at product rule $\quad x^2\tan y \to x^2\sec^2 y\frac{dy}{dx} + ...x\tan y$	M1
	Correct differentiation $\quad 2x\tan y + x^2\sec^2 y\frac{dy}{dx} + \frac{64y}{\pi^2}\frac{dy}{dx} = 0$	A1
	Substitutes $x = 3, y = \frac{\pi}{4} \Rightarrow 18\frac{dy}{dx} + 6 + \frac{16}{\pi}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3\pi}{8+9\pi}$	dM1, A1
		(5)
(c)	Calculates the gradient of the tangent at Q is $"\frac{3\pi}{8+9\pi}"$ o.e.	M1
	$\left(\text{grad of tan to } C \text{ at } P\right)\times\left(\text{grad of norm to } C \text{ at } Q\right) = -\frac{3\pi}{8+9\pi}\times -\frac{8+9\pi}{3\pi} = 1$	A1
		(2)
		(9 marks)

(a)

M1: Substitutes $y = \frac{\pi}{4}$ in equation of curve and proceeds to $x^2 = ...$ using $\tan\frac{\pi}{4} = 1$

This may be implied by $x^2 + ... = ... \Rightarrow x = ...$

A1: $x = \pm 3$ following the award of M1

(b)

M1: Attempts at chain rule $\quad y^2 \to ...y\frac{dy}{dx}$

M1: Attempt at product rule $\quad x^2\tan y \to x^2\sec^2 y\frac{dy}{dx} + ...x\tan y$

A1: Correct differentiation $x^2\sec^2 y\frac{dy}{dx} + 2x\tan y + \frac{64y}{\pi^2}\frac{dy}{dx} = 0$

You may see different forms such as $\pi^2x^2\sec^2 y\frac{dy}{dx} + 2\pi^2 x\tan y + 64y\frac{dy}{dx} = 0$ or

$x^2\sec^2 y dy + 2x\tan y dx + \frac{64y}{\pi^2}dy = 0$

Do not allow $\frac{dy}{dx} = x^2\sec^2 y\frac{dy}{dx} + 2x\tan y + \frac{64y}{\pi^2}\frac{dy}{dx}$ unless subsequent work is correct

M1: Substitutes $x = "3", y = \frac{\pi}{4}$ into a differentiated function and finds a value for $\frac{dy}{dx}$.

There must be exactly two $\frac{dy}{dx}$ terms, one from differentiating each of y^2 and $x^2 \tan y$

A1: $\frac{dy}{dx} = -\frac{3\pi}{8+9\pi}$ or exact equivalent such as $\frac{-6\pi}{16+18\pi}$ or $-\frac{6}{18+\frac{16}{\pi}}$

(c) Via calculation

M1: Substitutes $x = "-3", y = \frac{\pi}{4}$ into a differentiated function and finds a value for $\frac{dy}{dx}$ at Q.

It is a method that must be seen, via embedded values of $x = "-3", y = \frac{\pi}{4}$ or else implied by a correct value for their $x = "-3", y = \frac{\pi}{4}$ following the calculation of their $\frac{dy}{dx}$ at P.

You may see at attempt at $-\frac{dx}{dy}$ at Q using $x = "-3", y = \frac{\pi}{4}$

A1: Achieves the value 1 following correct and exact values for $\frac{dy}{dx}$ at P and $\frac{dy}{dx}$ or $-\frac{dx}{dy}$ at Q

……..

Alt (c) Via reasoning/deduction. To score via this method

- their $\frac{dy}{dx}$ must be of the correct form e.g. $\frac{dy}{dx} = \frac{Ax \tan y}{Bx^2 \sec^2 y + Cy}$
- and at P and Q, the y values are the same and the x values $\pm k$

M1: Subject to the above conditions, deduces that

'Gradient of tangent at P is m, gradient of tangent at Q is $-m$'

A1: Hence gradient of tangent at P × gradient of normal at Q is $m \times -\frac{1}{-m} = 1$ o.e , following M1

This can only be scored if $\frac{dy}{dx}$ is correct. If unsure send to review

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Question Number	Scheme	Marks
8 (a)	States/ uses $V=\lambda r^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$ OR $\dfrac{\mathrm{d}V}{\mathrm{d}t}=-C$	M1
	Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}r}\times\dfrac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow -C=\mu r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{C}{4\pi r^2}=-\dfrac{k}{r^2}$	A1*
		(3)
(b)	$\dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{k}{r^2} \Rightarrow \int r^2\mathrm{d}r=\int -k\ \mathrm{d}t$	
	$\dfrac{1}{3}r^3=-kt+c$	M1 A1
	Uses $t=0,\ r=30$ $\quad 9000=c$	M1
	Substitutes $r=12, t=24$ $\quad \dfrac{1}{3}\times 12^3=-k\times 24+9000 \Rightarrow k=(351)$	ddM1
	$r=\sqrt[3]{27000-1053t}$	A1
		(5)
(c)	$27000-1053T=0 \Rightarrow T=\ldots$	M1
	awrt 25.6	A1
		(2)
		(10 marks)

(a)

M1: States or uses either $\dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$ with any constant value for μ or any constant e.g. μ (allow any letter)

or $\dfrac{\mathrm{d}V}{\mathrm{d}t}=-C$ (Allow with k) but **must be a negative** and not given a value such as 1

If the formula for V is used (attempted) condone slips but it must obey $V=\lambda r^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$

dM1: Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}r}\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$ o.e. with $\dfrac{\mathrm{d}V}{\mathrm{d}t}=-C$ AND $\dfrac{\mathrm{d}V}{\mathrm{d}r}=\mu r^2$

$\Rightarrow -C=\mu r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$

See the first M1 for conditions on $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}V}{\mathrm{d}r}$

A1*: Proceeds to $\dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{k}{r^2}$ via a correct method. Look for $-C=4\pi r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow \dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{C}{4\pi r^2}=-\dfrac{k}{r^2}$

Condone a proof such as $-C=\mu r^2\times\dfrac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow \dfrac{\mathrm{d}r}{\mathrm{d}t}=-\dfrac{C}{\mu r^2}=-\dfrac{k}{r^2}$

If candidates had originally used k they must explain why $-\dfrac{k}{4\pi r^2}$ can be replaced by $-\dfrac{k}{r^2}$

If the formula for V is used it must be correct $V=\dfrac{4}{3}\pi r^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$