7.

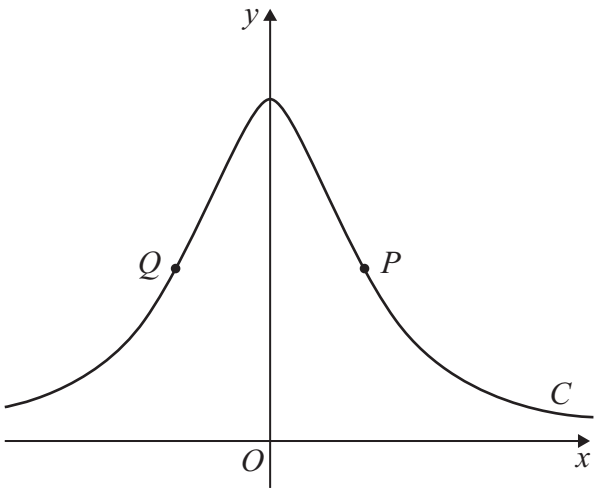


Figure 2

The curve C, shown in Figure 2 has equation

$$x^2 \tan y + \frac{32y^2}{\pi^2} = 11 \qquad 0 < y \leqslant \frac{\pi}{2}$$

The points P and Q, shown in Figure 2, lie on C and have y coordinate $\frac{\pi}{4}$

(a) Find the x coordinate of P and the x coordinate of Q.

(2)

(b) Use algebraic differentiation to find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P.

(5)

(c) Hence, showing your working, find the value of

(the gradient of the tangent to C at P) $\times$ (the gradient of the normal to C at Q)

(2)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

8. A large spherical balloon starts to deflate.

The balloon remains spherical whilst deflating.

At time t seconds after the balloon starts to deflate

- the balloon has radius r cm
- the balloon has volume V cm^3

Given that the volume of the balloon is modelled to decrease at a constant rate,

(a) use the model to show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 30 cm
- after 24 seconds the radius of the balloon is 12 cm

(b) solve the differential equation in part (a) to find an equation for r in terms of t.

(5)

Given that $V = 0$ when $t = T$

(c) use the model to find the value of T to one decimal place.

(2)

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