| Question Number | Scheme | Marks |
|---|---|---|
| **4 (a)** | Limits $a=5, b=10$ | B1 |
| | $\int_0^{\ln 6} \frac{2e^x+3}{e^x+4}\,dx = \int \frac{2\times(u-4)+3}{u}\times\frac{du}{(u-4)} = \int \frac{2u-5}{u(u-4)}\,du$ | M1 A1 |
| | | **(3)** |
| **(b)** | Sets their $\frac{2u-5}{u(u-4)}$ as $\frac{A}{u}+\frac{B}{u-4}$ | B1 |
| | $"2u-5" = A(u-4)+Bu \Rightarrow A=\frac{5}{4}, B=\frac{3}{4}$ | M1, A1 |
| | $\int_0^{\ln 6} \frac{2e^x+3}{e^x+4}\,dx = \int_5^{10} \frac{5/4}{u}+\frac{3/4}{(u-4)}\,du = \left[\frac{5}{4}\ln u+\frac{3}{4}\ln(u-4)\right]_5^{10}$ | M1 |
| | $=\left(\frac{5}{4}\ln 10+\frac{3}{4}\ln 6\right)-\left(\frac{5}{4}\ln 5+\frac{3}{4}\ln 1\right)$ | |
| | $=\frac{5}{4}\ln\frac{10}{5}+\frac{3}{4}\ln 6 = \frac{5}{4}\ln 2+\frac{3}{4}\ln 2+\frac{3}{4}\ln 3$ | dM1 |
| | $=2\ln 2+\frac{3}{4}\ln 3$ | A1 |
| | | **(6)** |
| | | **(9 marks)** |

(a)

B1: States either $a=5, b=10$ or correctly embeds the values as limits in the correct place within the integral. May be awarded from the limits seen or used in part (b)

M1: Attempts to write $\frac{2e^x+3}{e^x+4}\,dx$ in terms of $u$.

Look for both of

- A correct $\frac{dx}{du}$ or $\frac{du}{dx}$ in terms of $u$. E.g. $u=e^x+4 \Rightarrow \frac{du}{dx}=e^x=u-4$ or $x=\ln(u-4) \Rightarrow \frac{dx}{du}=\frac{1}{u-4}$
- $\frac{2e^x+3}{e^x+4}\,dx$ going to $\frac{cu+d}{u(u-4)}(du)$

A1: Shows that $\int \frac{2e^x+3}{e^x+4}\,dx = \int \frac{2u-5}{u(u-4)}\,du$ with the d$x$'s and d$u$'s appropriately placed throughout the solution. This mark does not depend on correct limits but the integral sign and d$u$ must present on the final line

(b)

B1: Scored for realising that this integral needs to be attempted via PF. So sets their $\dfrac{2u-5}{u(u-4)} \equiv \dfrac{A}{u}+\dfrac{B}{u-4}$

This may be implied by writing their expression in a form with numerical values for $A$ and $B$

M1: Valid attempt to find the values for $A$ and $B$.

Score for a correct identity followed by methods of substitution or equating terms to find values for $A$ and $B$. For example, sets $'cu+d=A(u-4)+Bu$ followed by $u=4 \Rightarrow B=...$ and $u=0 \Rightarrow A=...$

This can be implied by a correct value for $A$ or $B$

A1: Correct values for $A$ and $B$. This is not a follow through mark

Note that other methods of splitting the fraction are possible but the scheme can be applied.

FYI: $\dfrac{2u-5}{u(u-4)} \equiv \dfrac{2}{(u-4)}-\dfrac{5}{u(u-4)} \equiv \dfrac{2}{(u-4)}-\left\{-\dfrac{5}{4u}+\dfrac{5}{4(u-4)}\right\}$

M1: Integrates the form $\dfrac{A}{u}+\dfrac{B}{u-4}$ to $...\ln u+...\ln(u-4)$. Don't be concerned by the coefficients

Note that $...\ln pu+...\ln q(u-4)$ is also correct so you may see $\dfrac{5}{4u} \to \dfrac{5}{4}\ln 4u$

Condone missing brackets only if subsequent work involving limits implies this intention

dM1: Dependent upon the previous M. Scored for;

- Substituting 5 and 10 into $\alpha \ln u+\beta \ln(u-4)$ and subtracting either way around
- correctly applying at least one log law

A1: CAO $2\ln 2+\dfrac{3}{4}\ln 3$

| Question Number | Scheme | Marks |
|---|---|---|
| **5 (a)** | $(1,10)$ | B1, B1 **(2)** |
| **(b)** | $\frac{dy}{dt}=4t-4$, $\frac{dx}{dt}=\frac{4(2t+1)-2(4t-5)}{(2t+1)^2}$ | B1, M1 |
| | $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{(4t-4)}{14/(2t+1)^2}=\frac{2(t-1)(2t+1)^2}{7}$ | dM1 A1 **(4)** |
| **(c)** | Substitutes $t=3$ into their $\frac{dy}{dx}=(28)$ | |
| | $y-10=28(x-1)\Rightarrow y=28x-18$ | M1, A1 **(2)** |
| **(d) (i)** | $-5$,, $x<2$ | M1 A1 |
| **(ii)** | $f(x)\ldots 2$ | B1 **(3)** **(11 marks)** |

(a)

B1: One correct coordinate. E.g. $(1,...)$ or $(...,10)$. Allow as $x=1$ or $y=10$

B1: $(1,10)$. Allow as $x=1$, $y=10$

(b)

B1: $\frac{dy}{dt}=4t-4$. You may see this written as $y'=4t-4$ or $\dot{y}=2\times 2t-4$ which is fine.

M1: Attempts to use the quotient rule (or equivalent) to differentiate $x=\frac{4t-5}{2t+1}$.

Look for the form $\frac{dx}{dt}=\frac{p(2t+1)-q(4t-5)}{(2t+1)^2}$ with $p$, $q>0$

You may see $x=\frac{4t-5}{2t+1}\equiv 2-\frac{7}{2t+1}\Rightarrow\frac{dx}{dt}=\frac{14}{(2t+1)^2}$ with M1 scored for the form

$x=a+\frac{b}{2t+1}\Rightarrow\frac{dx}{dt}=\frac{c}{(2t+1)^2}$

Or the product rule $x=(4t-5)(2t+1)^{-1}\Rightarrow\frac{dx}{dt}=4(2t+1)^{-1}-2(4t-5)(2t+1)^{-2}$ but condone sign and coefficient slips. Look for the form $\alpha(2t+1)^{-1}\pm\beta(4t-5)(2t+1)^{-2}$