Question Number	Scheme	Marks
3 (a)	$(x+2y)(x-2y)$	B1 **(1)**
(b)	Makes one valid statement. Either $(x+2y)=27,(x-2y)=1$ or $(x+2y)=9,(x-2y)=3$	M1
	Solves correctly one of the pair; $(x+2y)=27,(x-2y)=1\Rightarrow x=14,y=6.5$	A1
	$(x+2y)=9,(x-2y)=3\Rightarrow x=6,y=1.5$	
	States and attempts to solve both valid equations	dM1
	Fully correct proof by contradiction (1) including correct set up, (2) correct solution of both equations and (3) correct conclusion*	A1* **(4)** **(5 marks)**

(a)

B1: $x^2-4y^2\equiv(x+2y)(x-2y)$

(b)

M1: Deduces either of the valid pairs of equations, $(x+2y)=27,(x-2y)=1$ or $(x+2y)=9,(x-2y)=3$.

There is no requirement to justify that $x+2y$ must be greater than $x-2y$ and therefore find the other equations $(x+2y)=1,(x-2y)=27$ and $(x+2y)=3,(x-2y)=9$

If they have part (a) incorrect, e.g $x^2-4y^2\equiv(x+4y)(x-4y)$ award for $(x+4y)=27,(x-4y)=1$ or $(x+4y)=9,(x-4y)=3$

A1: Correctly solves one of the two valid pairs of equations. No working is required, just the correct solution.

Look for either of $(x+2y)=27,(x-2y)=1\Rightarrow y=6.5,(x=14)$

Or $(x+2y)=9,(x-2y)=3\Rightarrow y=1.5,(x=6)$

Note that it is acceptable to only go as far as finding y as it is the non-integer solution

dM1: States and attempts to solve both valid pairs of equations. It is dependent upon the previous M mark

Note that it is acceptable to only go as far as finding y as that is the non-integer

You can **IGNORE** working from non-relevant equations like $(x+2y)=1,(x-2y)=27\Rightarrow x=...,y=...$ or $(x+2y)=-3,(x-2y)=9\Rightarrow x=...,y=...$ and incorrect equations like $(x+2y)=18,(x-2y)=1.5\Rightarrow x=...,y=...$

A1*: Full proof by contradiction. This must follow the award of M1, A1, dM1

Look for

- a correct set up with words such as 'assume there is' or 'let there be' with 'positive integers x and y' or 'positive integers'
- correct solution of both valid pair of equations
- acceptable (minimal) conclusion

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Minimal proof for 4 marks

Assume that there are positive integers x and y such that $x^2 - 4y^2 = 27$

So either $(x+2y) = 27, (x-2y) = 1 \Rightarrow y = 6.5$ ×

Or $(x+2y) = 9, (x-2y) = 3 \Rightarrow y = 1.5$ ×

Hence, we have a contradiction, so statement proven.

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Other methods exist.

For example:

There exist positive integers x and y such that $x^2 - 4y^2 = 27 \Rightarrow x^2 = 27 + 4y^2$

Then deduces that x^2 is odd which means that x is odd.

So $x = 2m+1 \Rightarrow (2m+1)^2 - 4y^2 = 27$

$$\Rightarrow 4m^2 + 4m - 4y^2 = 26$$

$$\Rightarrow y^2 = 4m^2 + 4m - \frac{13}{2}$$

Hence y^2 and so y cannot be an integer, so we have a contradiction,

Score M1: For correct work in deducing that x^2 or x is odd

A1: For stating that both x^2 and x are odd and x is of the form $2m+1$ o.e.

dM1: For squaring $2m+1$ and attempting to make y^2 the subject

A1: Full proof with correct set up, equations and deductions

Candidates who attempt odd and even combinations of x and y will only score marks when they reach an equivalent stage to the above. Please send these to review if you are unsure.

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Other more elaborate methods will be seen. If you feel that they deserve credit and you cannot see how to score the response, then please send to review.

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Question Number	Scheme	Marks
4 (a)	Limits $a=5, b=10$	B1
	$\int_0^{\ln 6} \frac{2e^x+3}{e^x+4}\,dx = \int \frac{2\times(u-4)+3}{u}\times\frac{du}{(u-4)} = \int \frac{2u-5}{u(u-4)}\,du$	M1 A1
		(3)
(b)	Sets their $\frac{2u-5}{u(u-4)}$ as $\frac{A}{u}+\frac{B}{u-4}$	B1
	$"2u-5" = A(u-4)+Bu \Rightarrow A=\frac{5}{4}, B=\frac{3}{4}$	M1, A1
	$\int_0^{\ln 6} \frac{2e^x+3}{e^x+4}\,dx = \int_5^{10} \frac{5/4}{u}+\frac{3/4}{(u-4)}\,du = \left[\frac{5}{4}\ln u+\frac{3}{4}\ln(u-4)\right]_5^{10}$	M1
	$=\left(\frac{5}{4}\ln 10+\frac{3}{4}\ln 6\right)-\left(\frac{5}{4}\ln 5+\frac{3}{4}\ln 1\right)$	
	$=\frac{5}{4}\ln\frac{10}{5}+\frac{3}{4}\ln 6 = \frac{5}{4}\ln 2+\frac{3}{4}\ln 2+\frac{3}{4}\ln 3$	dM1
	$=2\ln 2+\frac{3}{4}\ln 3$	A1
		(6)
		(9 marks)

(a)

B1: States either $a=5, b=10$ or correctly embeds the values as limits in the correct place within the integral. May be awarded from the limits seen or used in part (b)

M1: Attempts to write $\frac{2e^x+3}{e^x+4}\,dx$ in terms of u.

Look for both of

- A correct $\frac{dx}{du}$ or $\frac{du}{dx}$ in terms of u. E.g. $u=e^x+4 \Rightarrow \frac{du}{dx}=e^x=u-4$ or $x=\ln(u-4) \Rightarrow \frac{dx}{du}=\frac{1}{u-4}$
- $\frac{2e^x+3}{e^x+4}\,dx$ going to $\frac{cu+d}{u(u-4)}(du)$

A1: Shows that $\int \frac{2e^x+3}{e^x+4}\,dx = \int \frac{2u-5}{u(u-4)}\,du$ with the dx's and du's appropriately placed throughout the solution. This mark does not depend on correct limits but the integral sign and du must present on the final line