Question Number	Scheme	Marks
2	$\int x^3 \ln\left(\frac{1}{2}x\right) \mathrm{d}x = \frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \int \frac{1}{4}x^4 \times \frac{1}{x} \mathrm{d}x$ $= \frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \frac{1}{16}x^4 (+c)$ $\left[\frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \frac{1}{16}x^4\right]_2^{2\mathrm{e}^2} = \left(\frac{16\mathrm{e}^8}{4}\ln \mathrm{e}^2 - \frac{16\mathrm{e}^8}{16}\right) - \left(\frac{16}{4}\ln 1 - \frac{16}{16}\right)$ $= 7\mathrm{e}^8 + 1$	M1 dM1 A1 ddM1 A1 **(5 marks)**

Condone missing dx's throughout

M1: Attempts to integrate by parts the correct way around.

Look for $\int x^3 \ln\left(\frac{1}{2}x\right) \mathrm{d}x = ax^4 \ln\left(\frac{1}{2}x\right) \pm \int bx^4 \times \frac{1}{x} \mathrm{d}x$ o.e. such as $ax^4 \ln\left(\frac{1}{2}x\right) \pm \int bx^3 \mathrm{d}x$ with $a > 0$

dM1: And then integrates again to a form $px^4 \ln\left(\frac{1}{2}x\right) - qx^4$ where p and q are **positive constants**.

A1: Correct simplified or un-simplified integration. $\frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \frac{1}{16}x^4$ with or without the bracket

ddM1: Attempts to substitute in both limits and subtract either way around, condoning slips. It is dependent upon having an integral of the correct form. To score this mark

- Look for integration that leads to the form $px^4 \ln\left(\frac{1}{2}x\right) \pm qx^4$ where p and q are **positive constants**
- Substitution of both limits and an attempt to subtract either way around
- Correct calculation of both $\ln\left(\frac{1}{2} \times 2\mathrm{e}^2\right) = 2$ and $\ln\left(\frac{1}{2} \times 2\right) = 0$ which may be implied

A1: $7\mathrm{e}^8 + 1$ which must be in simplest form

……

Note: You may see D and I methods attempted. For the first three marks score M1, dM1 for integration to the form $px^4 \ln\left(\frac{1}{2}x\right) - qx^4$ where p and q are **positive constants.**

You may see something like this;

D	I
$\ln\left(\frac{1}{2}x\right)$	x^3
$\frac{1}{x}$	$\frac{x^4}{4}$

You could also see methods involving substitution in which the marks are scored as in the main scheme.

E.g. Let $u = \frac{x}{2}$ then $\int_2^{2\mathrm{e}^2} x^3 \ln\left(\frac{1}{2}x\right) \mathrm{d}x = \int_1^{\mathrm{e}^2} 16x^3 \ln u \, \mathrm{d}u = \left[4u^4 \ln u - u^4\right]_1^{\mathrm{e}^2} = 7\mathrm{e}^8 + 1$

……

Question Number	Scheme	Marks
3 (a)	$(x+2y)(x-2y)$	B1 **(1)**
(b)	Makes one valid statement. Either $(x+2y)=27,(x-2y)=1$ or $(x+2y)=9,(x-2y)=3$	M1
	Solves correctly one of the pair; $(x+2y)=27,(x-2y)=1\Rightarrow x=14,y=6.5$	A1
	$(x+2y)=9,(x-2y)=3\Rightarrow x=6,y=1.5$	
	States and attempts to solve both valid equations	dM1
	Fully correct proof by contradiction (1) including correct set up, (2) correct solution of both equations and (3) correct conclusion*	A1* **(4)** **(5 marks)**

(a)

B1: $x^2-4y^2\equiv(x+2y)(x-2y)$

(b)

M1: Deduces either of the valid pairs of equations, $(x+2y)=27,(x-2y)=1$ or $(x+2y)=9,(x-2y)=3$.

There is no requirement to justify that $x+2y$ must be greater than $x-2y$ and therefore find the other equations $(x+2y)=1,(x-2y)=27$ and $(x+2y)=3,(x-2y)=9$

If they have part (a) incorrect, e.g $x^2-4y^2\equiv(x+4y)(x-4y)$ award for $(x+4y)=27,(x-4y)=1$ or $(x+4y)=9,(x-4y)=3$

A1: Correctly solves one of the two valid pairs of equations. No working is required, just the correct solution.

Look for either of $(x+2y)=27,(x-2y)=1\Rightarrow y=6.5,(x=14)$

Or $(x+2y)=9,(x-2y)=3\Rightarrow y=1.5,(x=6)$

Note that it is acceptable to only go as far as finding y as it is the non-integer solution

dM1: States and attempts to solve both valid pairs of equations. It is dependent upon the previous M mark

Note that it is acceptable to only go as far as finding y as that is the non-integer

You can **IGNORE** working from non-relevant equations like $(x+2y)=1,(x-2y)=27\Rightarrow x=...,y=...$ or $(x+2y)=-3,(x-2y)=9\Rightarrow x=...,y=...$ and incorrect equations like $(x+2y)=18,(x-2y)=1.5\Rightarrow x=...,y=...$

A1*: Full proof by contradiction. This must follow the award of M1, A1, dM1

Look for

- a correct set up with words such as 'assume there is' or 'let there be' with 'positive integers x and y' or 'positive integers'
- correct solution of both valid pair of equations
- acceptable (minimal) conclusion