Question Number	Scheme	Marks
8(a)	A and B are where $y = 0$ so $t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3 \quad (0 \text{ and } -3)$ Substitutes $t = 3$ in $x = 3^2 + 2\times 3 = 15$ $B(15, 0)$ $A = (3, 0)$	M1 A1* B1 **(3)**
8(a) ALT	Uses answer $x = 15$: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5) Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ $B(15, 0)$ $A = (3, 0)$	M1 A1* B1 **(3)**
(b)	$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3t^2 - 9}{2t + 2}$ Substitutes $t = 3$ into $\dfrac{dy}{dx} = \dfrac{3t^2 - 9}{2t + 2} \Rightarrow \dfrac{dy}{dx} = \dfrac{9}{4}$ Uses their $\dfrac{9}{4}$ with $(15, 0)$ to produce tangent equation $9x - 4y - 135 = 0$	M1 A1 dM1 A1 **(4)**
(c)	Substitutes $x = t^2 + 2t$, $y = t^3 - 9t$, into their $9x - 4y - 135 = 0$ $\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$ $\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$ $\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$ $t = -\dfrac{15}{4}$ Coordinates of X are $\left(\dfrac{105}{16}, -\dfrac{1215}{64}\right)$	M1 dM1 A1 dM1A1 **(5)** **(12 marks)**

(a)

M1: States $t^3 - 9t = 0$ o.e. such as $y = 0 \Rightarrow t = 3$

No working needs to be seen for solving the equation and other values may be present including 0 and -3

A1*: Substitutes $t = 3$ in $x = t^2 + 2t$ to show that the x coordinate of B is 15 (and the y coordinate is 0)

This can be stated e.g. sub $t = 3$ in $x = t^2 + 2t \Rightarrow x = 15$

or else you need see some working such as 9 + 6, $3^2 + 2\times 3$ o.e

It is a given answer so **you must see evidence** followed by a statement *B* = (15, 0) o.e. such as *B* $x = 15, y = 0$

So just stating $t = 3$ followed by $x = 15$ and *B* = (15, 0) will score M1, A0*

B1: States $A = (3,0)$ (need not see working)

Alt (a)

C. M1: Sets $t^2 + 2t = 15 \Rightarrow t = 3$

A1*: Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ and makes a comment that *B* is on the *x*-axis

B1: States $A = (3,0)$ (need not see working)

(b)

M1: Attempts to find the value of $\frac{dy}{dx}$ at $t = \pm 3$

or *t* = – 5. $\frac{dy}{dx}$ must be attempted via $\frac{dy/dt}{dx/dt} = \frac{\text{Quadratic function}}{\text{Linear function}}$

A1: Achieves $\frac{dy}{dx}$ = 9/4 following M1

dM1: Uses (15,0) with the value of their gradient found using a correct method to find an equation
of the tangent.
It is dependent upon the previous M mark

A1: Achieves $9x - 4y - 135 = 0$ or any integer multiple of this

(c)

M1: Substitutes $x = t^2 + 2t,\ y = t^3 - 9t$ into their $9x - 4y - 135 = 0$ to form a simplified cubic equation in just *t*

dM1: This is dependent upon the previous M mark. It is for an attempt at using a non-calculator method to solve a cubic equation using the fact that $t = 3$ is a root. So, look for an attempt to divide their simplified cubic equation by (*t* – 3) or $t^2 - 6t + 9$

A1: Achieves $t = -\frac{15}{4}$ o.e. following correct cubic and correct factorisation. Look for prior working of

- $(t^2 - 6t + 9)(4t + 15) = 0,$ which may or may not be followed by $(t-3)^2(4t+15) = 0$
- $(t-3)(4t^2 + 3t - 45) = 0$ followed by a solution of the quadratic that could be via a calculator

Note that $4t^3 - 9t^2 - 54t + 135 = 0 \Rightarrow (t-3)^2(4t+15) = 0$ is dM0, A0 unless an explanation

is given alluding to the fact that $t=3$ is a double root, so $(t-3)^2$ is a factor, and so

$4t^3-9t^2-54t+135=(t-3)^2(At+B)=0$

giving $A = 4$ and $B = 15$ via inspection

dM1: Uses their value of t to find **either** the x or y co-ordinate of X.

It is dependent upon the FIRST M only and achieving a correct value of t for their equation.

It can be implied by a 'correct' value

A1: Coordinates of X are $\left(\frac{105}{16},-\frac{1215}{64}\right)$ or exact equivalent, for example, $x = 6.5625$, $y = -18.984375$

We are likely to see solutions without sight of the necessary steps required by the question

Solutions lacking necessary steps are scored as follows

Alternative 1; Via a simplified equation in t

Substitutes $x=t^2+2t,\ y=t^3-9t,$ into $9x-4y-135=0$

$\Rightarrow 9(t^2+2t)-4(t^3-9t)-135=0$

$\Rightarrow 4t^3-9t^2-54t+135=0 \Rightarrow t=-3.75$.

So, $X = \left(\frac{105}{16},-\frac{1215}{64}\right)$

Scores SC 1, 0, 0, 1, 1

M1: method shown

dM0: method not shown as presumably done via a calculator

A0: correct value for t but following dM0

dM1: For one correct coordinate (allowing either of $(6.56,-18.98)$ to 2dp) as it implies a correct method

A1: For exact answer $\left(\frac{105}{16},-\frac{1215}{64}\right)$

Alternative 2; Scores SC 0, 0, 0, 1, 1

No simplified equation in t

Substitutes $x=t^2+2t,\ y=t^3-9t,$ into $9x-4y-135=0$

$\Rightarrow 9(t^2+2t)-4(t^3-9t)-135=0$

$\Rightarrow t=-3.75$.

So $X = \left(\frac{105}{16},-\frac{1215}{64}\right)$ but allow $(6.56,-18.98)$ via this method

See Alternative 1 for the award of the final two marks

Question Number	Scheme	Marks
9(a)	$A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ Either $A: \lambda = 1$ or $B: \lambda = 3$ leading to $a = -3,\ b = 5$	M1 A1, A1 **(3)**
(b)	$\overrightarrow{AB} = = \pm\left(('5\mathbf{i}' - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ '- 3\mathbf{j}' + 5\mathbf{k})\right); = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	M1, A1 **(2)**
(c)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$ or $\overrightarrow{CA} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$	M1
	$\cos C\hat{A}B = \dfrac{\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2} . \sqrt{(3)^2 + (0)^2 + (-3)^2}}$	dM1
	$\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24} . \sqrt{18}} = \dfrac{\sqrt{3}}{2} \Rightarrow C\hat{A}B = 30^\circ$	A1 **(3)**
(d)	$\text{Area } CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ; = 3\sqrt{3}$	M1, A1 **(2)**
(e)	For one of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	M1, A1
	For both of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	M1, A1 **(4)** **(14 marks)**

(a)
M1: Finds the value of λ at *A* or *B* but is implied by a correct value for *a* or *b*
A1: At least one of *a* or *b* correct
A1: Both *a* and *b* correct

(b)
M1: Attempts to subtract the components either way around.