| Question Number | Scheme | Marks |
|---|---|---|
| **7(a)** | $\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)} \Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \ldots \text{ or } B = \ldots$ | M1 |
| | $A = -\frac{1}{2}, B = \frac{1}{2}$ giving $\frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$ | A1 |
| | | **[2]** |
| (b) | $\frac{\mathrm{d}x}{\mathrm{d}t} = k(4-x)(2-x),\ t \geqslant 0 \Rightarrow \int \frac{1}{(4-x)(2-x)}\,\mathrm{d}x = \int k\,\mathrm{d}t$ | |
| | $\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt\ (+c)$ | M1, A1 |
| | $\{t=0, x=0 \Rightarrow\}\ \frac{1}{2}\ln 4 - \frac{1}{2}\ln 2 = 0 + c\ \left\{\Rightarrow c = \frac{1}{2}\ln 2\right\}$ | M1 |
| | $\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt + \frac{1}{2}\ln 2 \Rightarrow \ln\left(\frac{4-x}{4-2x}\right) = 2kt$ | |
| | $\frac{4-x}{4-2x} = \mathrm{e}^{2kt}$ | M1 |
| | $4-x = 4\mathrm{e}^{2kt} - 2x\mathrm{e}^{2kt} \Rightarrow 4 - 4\mathrm{e}^{2kt} = x - 2x\mathrm{e}^{2kt}$ | |
| | $\Rightarrow 4 - 4\mathrm{e}^{2kt} = x(1 - 2\mathrm{e}^{2kt}) \Rightarrow x = \frac{4-4\mathrm{e}^{2kt}}{1-2\mathrm{e}^{2kt}}\ (*)$ | dM1, A1* |
| | | **[6]** |
| (c) | $1 = \frac{4-4\mathrm{e}^{0.2t}}{1-2\mathrm{e}^{0.2t}} \Rightarrow 2\mathrm{e}^{0.2t} = 3$ | M1 |
| | $\Rightarrow 0.2t = \ln\frac{3}{2} \Rightarrow t = \text{awrt } 2.03$ | A1 |
| | | **[2]** |
| | | **10 marks** |

(a)

M1: For forming a correct identity leading to one value of $A$ or $B$.

For example, $1 \equiv A(2-x) + B(4-x)$ from $\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)}$ and finds at least one of $A = \ldots$ or $B = \ldots$.

This would be implied using cover up rule and reaching one correct fraction

A1: $\frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$ or any equivalent form. **It is not scored for just the values**

ISW after sight of a correct answer. It may be awarded in part (b)

(b)

M1: Integrates to the form $\pm\lambda\ln(4-x) \pm \mu\ln(2-x),\ \lambda \neq 0, \mu \neq 0$

Note that versions such as $\pm\lambda\ln(8-2x) \pm \mu\ln(4-2x),\ \lambda \neq 0, \mu \neq 0$ ARE correct

A1: Fully correct integration of both left and right-hand side condoning a missing constant of integration.

Look for $\frac{1}{2}\ln(4-x)-\frac{1}{2}\ln(2-x)=kt$ o.e. such as $\frac{1}{2k}\left(\ln(4-x)-\ln(2-x)\right)=t$

Note that you may see $\frac{-\frac{1}{2}}{(4-x)}$ being integrated to $\frac{1}{2}\ln(8-2x)$

Similarly, $\frac{\frac{1}{2}}{(2-x)}$ being integrated to $-\frac{1}{2}\ln(4-2x)$. These are both correct

M1: Using both $t=0$ and $x=0$ in an integrated equation containing a constant of integration and finds the value of *c*. If the *k* is 'moved' to the LHS, *c* will be found in terms of *k*.
It is dependent upon having integrated to a form equivalent to
$\pm\lambda\ln(4-x)\pm\mu\ln(2-x)=f(t)+c$ o.e.

M1: Starting from an equation of the form $\pm\lambda\ln(4-x)\pm\mu\ln(2-x)=\pm kt+c$, $\lambda$, $\mu$, $k$, $c\neq 0$ o.e. and applies a fully correct method to eliminate their logarithms. To score this

- the lns must be correctly combined
- all exponential work must be correct at the point when the logarithms are removed

For example, $\ln(4-x)-\ln(2-x)=kt+\ln 2\Rightarrow(4-x)-(2-x)=e^{kt+\ln 2}$ is M0

AND $\ln(4-x)-\ln(2-x)=kt+\ln 2\Rightarrow\ln\frac{(4-x)}{(2-x)}=kt+\ln 2\Rightarrow\frac{(4-x)}{(2-x)}=e^{kt+\ln 2}$ is M1 BUT $\frac{(4-x)}{(2-x)}=e^{kt}+2$ is M0

**There must be a constant of integration but it need not have been evaluated.**
**(See below for this type of solution)**

dM1: D**ependent on the previous M mark.** A complete method of rearranging to make *x* the subject. It can be awarded when the two terms in *x* are on one side of the equation and the terms independent of *x* are on the other.
So, following the award of the previous mark, look for

- cross multiplication leading to two terms in *x* and two terms independent of *x*
- collecting two *x* terms one side of the equation and two terms independent of *x* on the other

**There must be a constant of integration but it need not have been evaluated.**
**(See below for this type of solution)**

A1*: CSO. **Note that this is a given answer and all steps should be seen before awarding this mark.**

Condone a line where a bracket is missed e.g. $\ln 2-x \leftrightarrow \ln(2-x)$ as long as it is replaced in subsequent lines

If a candidate reaches $x-2xe^{2kt}=4-4e^{2kt}$ it is acceptable to just write down $x=\dfrac{4-4e^{2kt}}{1-2e^{2kt}}$

However, $2xe^{2kt}-x=4e^{2kt}-4$ must proceed to the given answer via $x=\dfrac{4e^{2kt}-4}{2e^{2kt}-1}$ o.e. such as $(2e^{2kt}-1)x=4e^{2kt}-4$ to score the A1*

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Alt (b) and the order of scoring the marks

M1, A1: $\dfrac{1}{2}\ln(4-x)-\dfrac{1}{2}\ln(2-x)=kt+c$

$\ln(4-x)-\ln(2-x)=2kt+C$

$\ln\dfrac{(4-x)}{(2-x)}=2kt+C\Rightarrow\dfrac{(4-x)}{(2-x)}=e^{2kt+C}$ ← 3rd M1 scored at this stage

As $e^{2kt+C}=e^{2kt}\times e^{C}=Ae^{2kt}$

$(4-x)=(2-x)Ae^{2kt}\Rightarrow Axe^{2kt}-x=2Ae^{2kt}-4\Rightarrow x=\dfrac{2Ae^{2kt}-4}{Ae^{2kt}-1}$ 4th M1 scored at this stage

Uses $t=0,\ x=0\ \Rightarrow 0=\dfrac{2A-4}{A-1}\Rightarrow A=2$ ← 2nd M1 scored at this stage

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(c)

M1: Substitutes $x=1$ and $k=0.1$ into the given equation in part (c) and proceeds to $pe^{0.2t}=q,\ pq>0$

A1: Proceeds to awrt $2.03$ (Note that M1 must have been awarded to award this mark)

Alternatives exist via $\dfrac{4-x}{4-2x}=e^{2kt}$ but essentially the same method applies

SC: Answers without (sufficient) working

– there may be more space than you need.

- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

An answer of 2.03 with little or no valid working meriting the M1 mark can score SC 1,0

| Question Number | Scheme | Marks |
|---|---|---|
| **8(a)** | $A$ and $B$ are where $y = 0$ so<br>$t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3 \quad (0 \text{ and } -3)$<br>Substitutes $t = 3$ in $x = 3^2 + 2\times 3 = 15$ $B(15, 0)$<br>$A = (3, 0)$ | M1<br>A1*<br>B1<br>**(3)** |
| **8(a) ALT** | Uses answer $x = 15$: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5)<br>Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ $B(15, 0)$<br>$A = (3, 0)$ | M1<br>A1*<br>B1<br>**(3)** |
| **(b)** | $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3t^2 - 9}{2t + 2}$<br>Substitutes $t = 3$ into $\dfrac{dy}{dx} = \dfrac{3t^2 - 9}{2t + 2} \Rightarrow \dfrac{dy}{dx} = \dfrac{9}{4}$<br>Uses their $\dfrac{9}{4}$ with $(15, 0)$ to produce tangent equation<br>$9x - 4y - 135 = 0$ | M1 A1<br>dM1 A1<br>**(4)** |
| **(c)** | Substitutes $x = t^2 + 2t$, $y = t^3 - 9t$, into their $9x - 4y - 135 = 0$<br>$\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$<br>$\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$<br>$\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$<br>$t = -\dfrac{15}{4}$<br>Coordinates of $X$ are $\left(\dfrac{105}{16}, -\dfrac{1215}{64}\right)$ | M1<br>dM1<br>A1<br>dM1A1<br>**(5)**<br>**(12 marks)** |

(a)

M1: States $t^3 - 9t = 0$ o.e. such as $y = 0 \Rightarrow t = 3$

No working needs to be seen for solving the equation and other values may be present including 0 and $-3$

A1*: Substitutes $t = 3$ in $x = t^2 + 2t$ to show that the $x$ coordinate of $B$ is 15 (and the $y$ coordinate is 0)

This can be stated e.g. sub $t = 3$ in $x = t^2 + 2t \Rightarrow x = 15$

or else you need see some working such as 9 + 6, $3^2 + 2\times 3$ o.e