7. (a) Express $\dfrac{1}{(4-x)(2-x)}$ in partial fractions.

(2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(4-x)(2-x) \qquad t \geqslant 0 \qquad 0 \leqslant x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

(b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4\mathrm{e}^{2kt}}{1 - 2\mathrm{e}^{2kt}}$$

(*Solutions relying on calculator technology are not acceptable.*)

(6)

Given that $k = 0.1$

(c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures.

(2)

(Total for Question 7 is 10 marks)

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

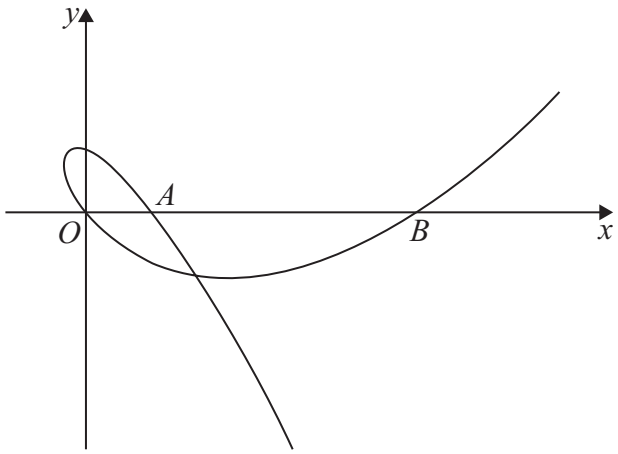


Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \qquad y = t^3 - 9t \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Find the equation of the tangent to the curve at B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The tangent to the curve at B cuts the curve again at point X.

(c) Use algebra to find the coordinates of X, showing each stage of your working.

(5)

(Total for Question 8 is 12 marks)