6.

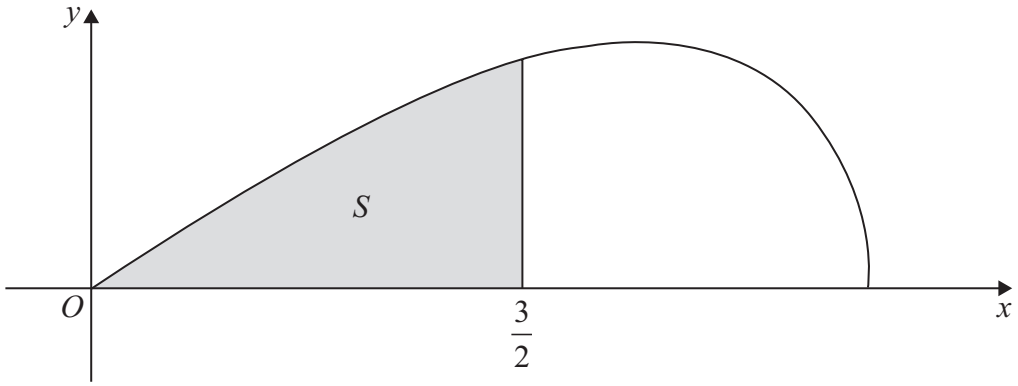


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 3\sin t \qquad y = 2\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The finite region S, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with the equation $x = \frac{3}{2}$

The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution is given by

$$k\int_0^a \sin^2 t \cos^3 t \, \mathrm{d}t$$

where k and a are constants to be given in terms of π.

(4)

(b) Use the substitution $u = \sin t$, or otherwise, to find the exact value of this volume, giving your answer in the form $\frac{p\pi}{q}$ where p and q are integers.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

(Total for Question 6 is 9 marks)

7. (a) Express $\dfrac{1}{(4-x)(2-x)}$ in partial fractions.

(2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(4-x)(2-x) \qquad t \geqslant 0 \qquad 0 \leqslant x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

(b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4\mathrm{e}^{2kt}}{1 - 2\mathrm{e}^{2kt}}$$

(*Solutions relying on calculator technology are not acceptable.*)

(6)

Given that $k = 0.1$

(c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures.

(2)

(Total for Question 7 is 10 marks)