4. (a) Use the binomial expansion, in ascending powers of x, of $\frac{1}{\sqrt{(1-2x)}}$ to show that

$$\frac{2+3x}{\sqrt{(1-2x)}} \approx 2 + 5x + 6x^2 \qquad |x| < 0.5$$

(4)

(b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{(1-2x)}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

(Total for Question 4 is 7 marks)

5.

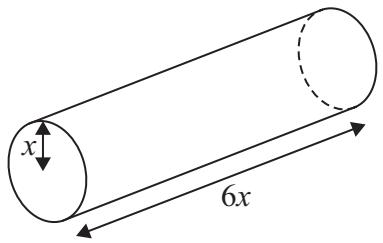


Figure 1

Figure 1 shows a right circular cylindrical rod which is expanding as it is heated.

At time t seconds the radius of the rod is x cm and the length of the rod is $6x$ cm.

Given that the **cross-sectional area** of the rod is increasing at a constant rate of $\frac{\pi}{20}$ cm^2 s^{-1}, find the rate of increase of the volume of the rod when $x = 2$

Write your answer in the form $k\pi$ cm^3 s^{-1} where k is a rational number.

(6)

(Total for Question 5 is 6 marks)

6.

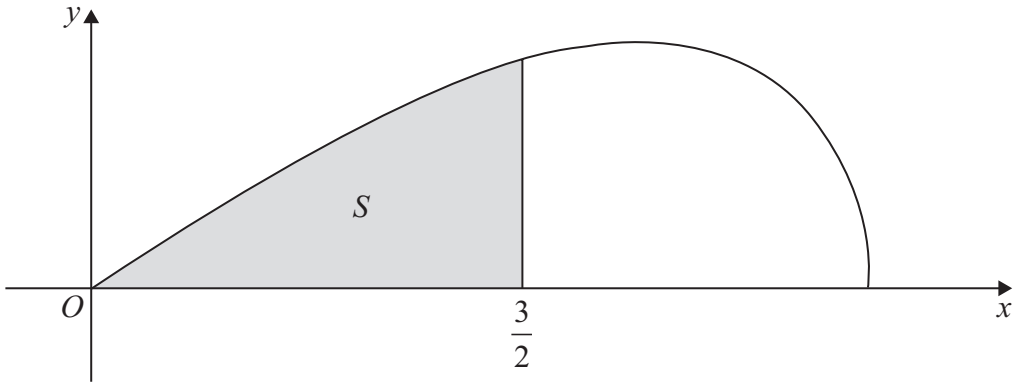


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 3\sin t \qquad y = 2\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The finite region S, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with the equation $x = \frac{3}{2}$

The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution is given by

$$k\int_0^a \sin^2 t \cos^3 t \, \mathrm{d}t$$

where k and a are constants to be given in terms of π.

(4)

(b) Use the substitution $u = \sin t$, or otherwise, to find the exact value of this volume, giving your answer in the form $\frac{p\pi}{q}$ where p and q are integers.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

(Total for Question 6 is 9 marks)