1. Prove by contradiction that, if a and b are real numbers, $a > 0$ and $b > 0$, then

$$\frac{9a}{b} + \frac{4b}{a} \geqslant 12$$

(4)

(Total for Question 1 is 4 marks)

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2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use integration by parts to find the exact value of $\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\,\mathrm{d}x$

Write your answer in the form $a + \frac{b}{\mathrm{e}}$, where a and b are integers.

(6)

(Total for Question 2 is 6 marks)

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3. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

(Total for Question 3 is 7 marks)

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4. (a) Use the binomial expansion, in ascending powers of x, of $\frac{1}{\sqrt{(1-2x)}}$ to show that

$$\frac{2+3x}{\sqrt{(1-2x)}} \approx 2 + 5x + 6x^2 \qquad |x| < 0.5$$

(4)

(b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{(1-2x)}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

(Total for Question 4 is 7 marks)

5.

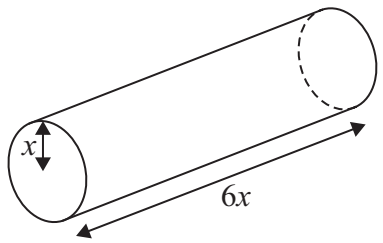


Figure 1

Figure 1 shows a right circular cylindrical rod which is expanding as it is heated.

At time t seconds the radius of the rod is x cm and the length of the rod is $6x$ cm.

Given that the **cross-sectional area** of the rod is increasing at a constant rate of $\frac{\pi}{20}$ cm^2 s^{-1}, find the rate of increase of the volume of the rod when $x = 2$

Write your answer in the form $k\pi$ cm^3 s^{-1} where k is a rational number.

(6)

(Total for Question 5 is 6 marks)
