1. Prove by contradiction that, if a and b are real numbers, $a > 0$ and $b > 0$, then

$$\frac{9a}{b} + \frac{4b}{a} \geqslant 12$$

(4)

(Total for Question 1 is 4 marks)

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2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use integration by parts to find the exact value of $\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\,\mathrm{d}x$

Write your answer in the form $a + \frac{b}{\mathrm{e}}$, where a and b are integers.

(6)

(Total for Question 2 is 6 marks)

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3. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

(Total for Question 3 is 7 marks)
