9.

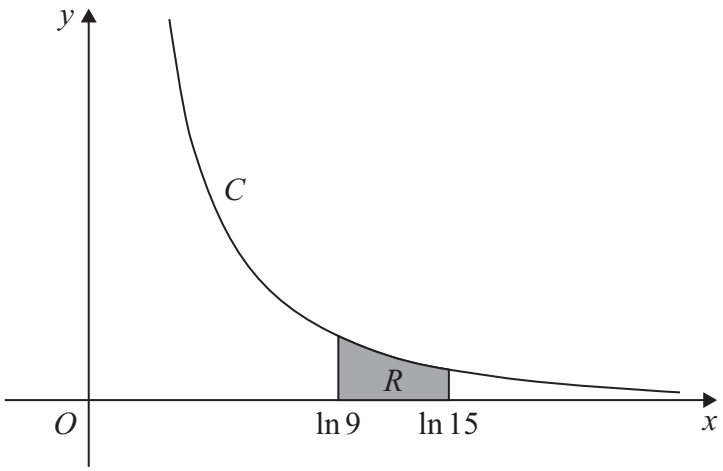

Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t + 1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P.

(6)

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = \ln 9$, the x-axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t + 1)(2t + 5)}\,\mathrm{d}t$$

where a, b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R.

Write your answer in the form $\frac{1}{3}\ln\alpha$, where α is a rational number.

(9)

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 15 marks)

10. (a) Statement X: "If x^2 is an odd square number, then x is an odd number."

A student was asked to prove Statement X using proof by contradiction.

The student begins by writing the following.

> Assume that x^2 is an odd square number and x is an even number.
>
> If x is an even number it can be expressed in the form $x = 2m \quad m \in \mathbb{N}$

Complete the proof.

(2)

(b) Use proof by contradiction to show that

"There are no integers a and b such that $a^2 - 4b = 27$"

You may assume Statement X without further justification.

(4)

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