8. A container initially contains 6 litres of water.

Water starts to flow into the container.

The rate at which the water flows into the container is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 12t\mathrm{e}^{-t}$$

where V litres is the volume of water in the container, t minutes after water starts flowing into the container.

(a) Solve the differential equation and use the initial condition to find an equation for V in terms of t

(5)

Given that the capacity of the container is 20 litres,

(b) determine, according to the model, whether or not the container will ever become full, giving a reason for your answer.

(1)

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Question 8 continued

(Total for Question 8 is 6 marks)

9.

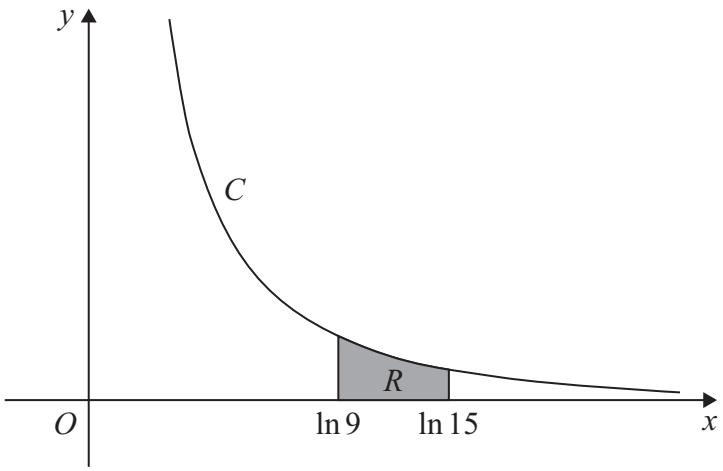

Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t + 1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P.

(6)

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = \ln 9$, the x-axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t + 1)(2t + 5)}\,\mathrm{d}t$$

where a, b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R.

Write your answer in the form $\frac{1}{3}\ln\alpha$, where α is a rational number.

(9)

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