7.

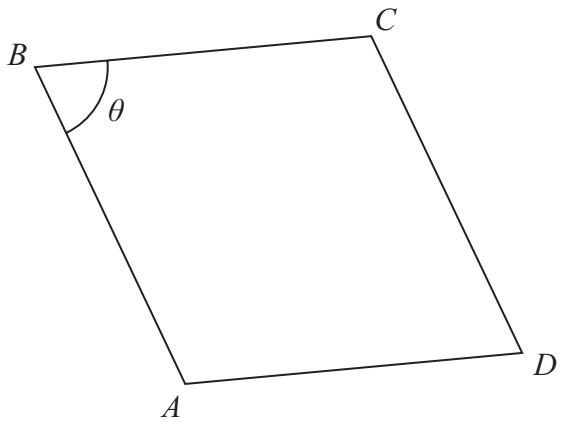

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\overrightarrow{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90°$

(a) find the exact value of $\cos\theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

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Question 7 continued

(Total for Question 7 is 6 marks)

8. A container initially contains 6 litres of water.

Water starts to flow into the container.

The rate at which the water flows into the container is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 12t\mathrm{e}^{-t}$$

where V litres is the volume of water in the container, t minutes after water starts flowing into the container.

(a) Solve the differential equation and use the initial condition to find an equation for V in terms of t

(5)

Given that the capacity of the container is 20 litres,

(b) determine, according to the model, whether or not the container will ever become full, giving a reason for your answer.

(1)

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