Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt}=\frac{-1}{(t+1)^2}$, $\frac{dx}{dt}=\frac{2}{2t+5}$ $\Rightarrow\left(\frac{dy}{dx}=\right)-\frac{2t+5}{2(t+1)^2}$	M1A1
	Sets $-\frac{2t+5}{2(t+1)^2}=-4\Rightarrow 8t^2+14t+3=0$	M1A1
	$\Rightarrow(4t+1)(2t+3)=0\Rightarrow t=-\frac{1}{4}\Rightarrow(y=)\ \frac{4}{3}$	dM1A1
		(6)
(b) (i)	$a=2, b=5$	B1
	Area $=\int y\frac{dx}{dt}\,dt=\int_2^5\frac{1}{t+1}\times\frac{2}{2t+5}\,dt=\int_2^5\frac{2}{(t+1)(2t+5)}\,dt$	M1A1
(ii)	$\frac{2}{(t+1)(2t+5)}\equiv\frac{A}{(t+1)}+\frac{B}{(2t+5)}\Rightarrow A=...,\ B=...$	M1
	$=\frac{2/3}{(t+1)}-\frac{4/3}{(2t+5)}$	A1
	$\int\frac{2/3}{(t+1)}-\frac{4/3}{(2t+5)}\,dt=\frac{2}{3}\ln\lvert t+1\rvert-\frac{2}{3}\ln\lvert 2t+5\rvert$	M1A1ft
	$\int_2^5\frac{2}{(t+1)(2t+5)}\,dt=\left(\frac{2}{3}\ln 6-\frac{2}{3}\ln 15\right)-\left(\frac{2}{3}\ln 3-\frac{2}{3}\ln 9\right)$ $=\frac{2}{3}\ln\left(\frac{6}{5}\right)=\frac{1}{3}\ln\left(\frac{36}{25}\right)$	dM1A1
		(9) **(15 marks)**

(a)

M1: Attempts to find $\frac{dy}{dx}$ by dividing their $\frac{dy}{dt}$ by their $\frac{dx}{dt}$. Both must be attempts at differentiation, however poor. Do not allow e.g. $\frac{2}{2t+5}\rightarrow...\ln(2t+5)$ as this is clearly integration.

A1: $\left(\frac{dy}{dx}=\right)-\frac{2t+5}{2(t+1)^2}$ which may be unsimplified and may be implied by further work

M1: Sets their "$-\frac{2t+5}{2(t+1)^2}$" $=\pm 4$ and proceeds to a 3TQ in t. Do not be concerned by the mechanics of the rearrangement.

A1: $8t^2+14t+3=0$ o.e. e.g. $t^2+\frac{7}{4}t+\frac{3}{8}=0$ The = 0 may be implied by further work if the three terms are on the same side of the equation.

dM1: Dependent upon both previous M's, score for solving their 3TQ AND finding a value for *y*. Allow any valid attempt to solving the quadratic including via a calculator. You may need to check this if just a value for *t* is stated. Condone a *y* value being found from a value of *t* such that $t<-1$.

A1: $(y=)\ \frac{4}{3}$ oe only (ignore the *x* coordinate if found)

See alt below – send to review if unsure

9 (a)	$y=\frac{2}{e^x-3} \Rightarrow \frac{dy}{dx}=\frac{-2e^x}{(e^x-3)^2}$	M1A1
	Sets $\frac{-2e^x}{(e^x-3)^2}=-4 \Rightarrow 2(e^x)^2-13e^x+18=0=0$	M1A1
	$\Rightarrow \left(2e^x-9\right)\left(e^x-2\right)=0 \Rightarrow e^x=\frac{9}{2} \Rightarrow t=-\frac{1}{4} \Rightarrow y=\frac{4}{3}$	dM1A1
		(6)

(b)(i)

B1: Correct values for *a* and *b*. Must be correctly paired or indicated on the integral. If there is a contradiction between the two then the values on the integral sign take precedence.

M1: For attempting $y\frac{dx}{dt}$ with their $\frac{dx}{dt}$ in the form $\frac{k}{2t+5}$ where *k* is a constant.

Must see either the general expression $y\frac{dx}{dt}$ stated or written as e.g. $\frac{1}{t+1}\times\frac{k}{2t+5}$ oe. There needs to be an intermediate stage of working before writing $\frac{a}{(t+1)(2t+5)}$.

It cannot be implied by $\int_a^b \frac{2}{(t+1)(2t+5)}\,\mathrm{d}t$

A1: Area $R = \int_a^b \frac{2}{(t+1)(2t+5)}\,\mathrm{d}t$ with or without correct values for a and b. **The dt only needs to be present in the final answer.** Allow if seen at the start of (b)(ii). Allow slips in working to be recovered provided it is before the final answer. This mark cannot be scored without the previous M mark being scored i.e. we need to see an intermediate stage of working.

Note allow $\int_a^b \frac{-2}{(t+1)(2t+5)}\,\mathrm{d}t$ only if their limits are clearly stated as being the other way round.

(b)(ii)

M1: Attempts to write $\frac{k}{(t+1)(2t+5)}$ as $\frac{A}{(t+1)}+\frac{B}{(2t+5)}$. May be implied by a correct value for A or B (but not e.g. $A=1,\ B=1$ without a valid method seen) May have taken a factor outside of the integral.

A1: $\frac{2}{(t+1)(2t+5)} \equiv \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)}$. May have taken a factor outside of the integral so allow e.g. $\frac{1/3}{(t+1)} - \frac{2/3}{(2t+5)}$ oe to score. Must see the partial fractions written or implied by their integration.

M1: Correct attempt at integration. Look for $\pm\alpha\ln|t+1| \pm\beta\ln|2t+5|$ oe where $\alpha,\ \beta \neq 0$ They must have separated into partial fractions first. Condone brackets instead of moduli and condone the omission of brackets to be implied by further work. Allow this mark to be scored if they miscopy their partial fractions (e.g. a sign slip or a coefficient slip) so they will still integrate to a similar form.

A1ft: $\int \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)}\,\mathrm{d}t = \frac{2}{3}\ln|t+1| - \frac{2}{3}\ln|2t+5|$ oe so allow $\frac{1}{3}\ln|t+1| - \frac{1}{3}\ln|2t+5|$ oe but ft on their *A* and *B*. May have taken a factor of 2 outside of the integral. Condone brackets instead of moduli and invisible brackets to be implied by further work. Ignore any constant of integration if present.

dM1: Attempts to use their limits and subtract either way round. They must use at least one law to combine at least two terms. They cannot proceed directly to the final answer. It is dependent on the previous method mark.

A1: $\frac{1}{3}\ln\left(\frac{36}{25}\right)$ o.e. in the required form $\frac{1}{3}\ln a$ where *a* is a rational number. Do not accept $\frac{1}{3}\ln\left(\frac{6}{5}\right)^2$

Question Number	Scheme	Marks
10 (a)	Requires a minimum of • $x^2=(2m)^2=4m^2$ • which is even so contradiction, so proven	 B1 dB1 **(2)**
(b)	e.g. $a^2-4b=27\Rightarrow a^2=27+4b$ which is odd e.g. If a^2 is odd then *a* is odd and can be written in the form $a=2m+1$ e.g. $(2m+1)^2=27+4b\Rightarrow b=...$ Requires a full explanation and conclusion e.g. • $b=m^2+m-\frac{13}{2}$ which is not an integer • so contradiction, hence proven *	M1 A1 dM1 A1* **(4)** **(6 marks)**

(a)

B1: States that $x^2=4m^2$

dB1: Dependent on the previous B mark. Explains or concludes that $4m^2$ is **even** (or condone not odd) and either states contradiction/so proven or rewrites statement X (condone omission of some of the words provided the general conclusion is clear). Do not allow if the contradiction is not based on the assumption that x^2 is odd and $4m^2$ is even.

(b) Note that there many ways of showing this – send to review if you think it might be credit worthy

M1: $a^2-4b=27\Rightarrow a^2=27+4b$ oe with an indication that this is odd. Allow writing in the form $2(2b+13)+1$ to imply this.

A1: **States** a^2 is odd (then *a* is odd) and writes in the form $a=2m+1$

dM1: Attempts $(2m+1)^2=27+4b$ and rearranges to $b=...$ or $2b=...$ or a form where a contradiction can be deduced such as $m^2+m=b+\frac{13}{2}$ where m^2 and *m* will both be integers but $b+\frac{13}{2}$ is not. It is dependent on the