(b)	$\overrightarrow{CA} = (2-p)\mathbf{i} - 5\mathbf{j} + 6\mathbf{k} \qquad \overrightarrow{CB} = (3-p)\mathbf{i} - 5\mathbf{k}$	M1
	$(2-p)(3-p) - 30 = 0$	**d**M1
	$\Rightarrow 6 - 5p + p^2 - 30 = 0 \Rightarrow p^2 - 5p - 24 = 0 \Rightarrow p = ...$	**dd**M1
	$p = -3, \quad p = 8$	A1
		(4)
(c)	$\left\|\overrightarrow{CA}\right\| = \sqrt{(2-8)^2 + (-5)^2 + 6^2}$ or $\left\|\overrightarrow{CB}\right\| = \sqrt{(3-8)^2 + (-5)^2}$	M1
	$\left\|\overrightarrow{CA}\right\| = \sqrt{97}$ or $\left\|\overrightarrow{CB}\right\| = \sqrt{50}$	A1
	Area of triangle $= \frac{1}{2} \times \left\|\overrightarrow{CA}\right\| \times \left\|\overrightarrow{CB}\right\| = \frac{1}{2} \times \sqrt{50} \times \sqrt{97} =$ awrt 34.8 units^2 or e.g. $\frac{1}{2} \times \left\|\overrightarrow{CA}\right\| \times \left\|\overrightarrow{AB}\right\| \sin BAC = \frac{1}{2} \times \sqrt{50} \times \sqrt{147} \sin\left(\cos^{-1}\frac{\sqrt{50}}{\sqrt{147}}\right) =$ awrt 34.8 units^2	**d**M1A1
		(4)
		(11 marks)

(b) **Allow all marks in (b) for correct work using** $\pm\overrightarrow{CA}$ and/or $\pm\overrightarrow{CB}$

M1: Attempts to find either $\pm\overrightarrow{CA}$ or $\pm\overrightarrow{CB}$ using subtraction.
If no working is shown, it may be implied by 2 correct components.

dM1: **Depends on the previous method mark.**
Attempts both $\pm\overrightarrow{CA}$ **and** $\pm\overrightarrow{CB}$ using subtraction and attempts the scalar product between their $\pm\overrightarrow{CA}$ and their $\pm\overrightarrow{CB}$ **and** sets = 0 to obtain $\pm(2-p) \times \pm(3-p) \pm k = 0$ where $k \neq 0$

ddM1: **Depends on both previous method marks.**
Multiplies out, collects terms and attempts to solve their three term quadratic (usual rules apply – see general guidance). May be implied by correct roots for their 3TQ.
Do not condone work where they lose or ignore their *k*.

A1: Correct values $p = -3, \quad p = 8$

(c)

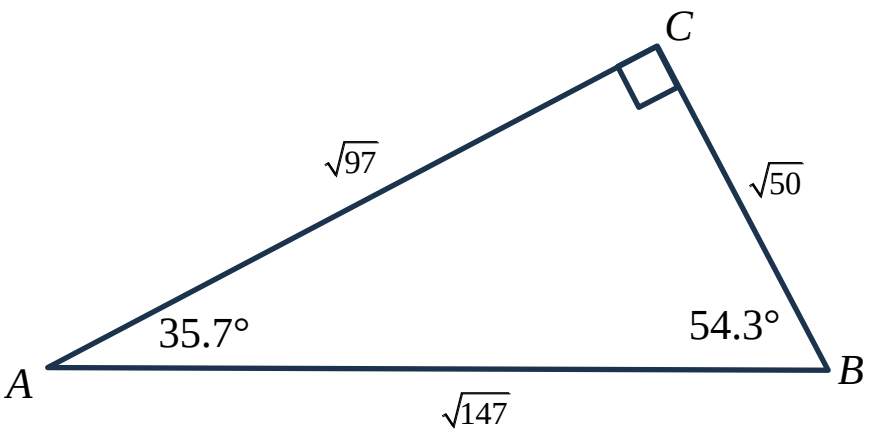
M1: Uses at least one of their values of *p* from part (b) (which may be negative) and attempts $\left|\overrightarrow{CA}\right|$ **or** $\left|\overrightarrow{CB}\right|$ using Pythagoras. May be implied.

A1: For $\left|\overrightarrow{CA}\right| = \sqrt{97}$ or $\left|\overrightarrow{CB}\right| = \sqrt{50}$ seen or implied.

dM1: **Depends on the previous method mark.**
Attempts to find the area of the triangle using a correct method e.g. $\frac{1}{2} \times$ their $\left|\overrightarrow{CA}\right| \times$ their $\left|\overrightarrow{CB}\right|$
It is dependent on the previous method mark and it is dependent on having used a **positive** value for *p*.
If they use a less direct method e.g. $\frac{1}{2} \times \left|\overrightarrow{CA}\right| \times \left|\overrightarrow{AB}\right| \sin BAC$ then it must be a correct method, including an attempt to find $\left|\overrightarrow{AB}\right|$ using Pythagoras.

A1: awrt 34.8 This answer and no others. Do not allow for exact answers.
If they use the other value of *p* and give another area which is not rejected, score A0

For reference:



Note that other methods may be possible in (c) for the area e.g. using a vector product:

$$p = 8 \Rightarrow \overrightarrow{CA} = -6\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}, \quad \overrightarrow{CB} = -5\mathbf{i} - 5\mathbf{k}$$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -5 & 6 \\ -5 & 0 & -5 \end{vmatrix} = \begin{pmatrix} 25 \\ -60 \\ -25 \end{pmatrix}$$

$$\text{Area of triangle} = \frac{1}{2}\sqrt{25^2 + 60^2 + 25^2} = 34.8$$

Score as:

M1: Attempts the vector product between 2 appropriate vectors e.g. $\overrightarrow{CA}$ and $\overrightarrow{CB}$
May be implied by 2 correct components.

A1: Correct vector

dM1: Attempts $\frac{1}{2} \times$ the magnitude of their vector

A1: For awrt 34.8

Question Number	Scheme	Marks
9(a)	Volume $=144\pi\int_0^{20}\frac{2x+3}{x^2+3x+5}\,\mathrm{d}x$ or e.g. $\pi\int_0^{20}\left(12\left(\frac{2x+3}{x^2+3x+5}\right)^{\frac{1}{2}}\right)^2\,\mathrm{d}x$	B1
	$144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{20}$	M1
	$=144\pi\left(\ln 465-\ln 5\right)=144\pi\ln 93\,(\text{units}^3)$	**d**M1A1
		(4)

(a)

B1: Correct integral for the volume of R which may be simplified or unsimplified.
The 144 (or 12^2) and the π and the limits 0 and 20 must be present or implied by subsequent work. Condone the omission of "dx".
$2\pi\times 144\int_0^{20}\frac{2x+3}{x^2+3x+5}\,\mathrm{d}x$ scores B0

M1: Integrates $\frac{2x+3}{x^2+3x+5}$ to obtain $...\ln\left(x^2+3x+5\right)$
Condone missing brackets e.g. $...\ln x^2+3x+5$
Condone poor notation e.g. $144\pi\int\frac{2x+3}{x^2+3x+5}\mathrm{d}x=\int 144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{20}\mathrm{d}x$
This may be implied by substitution e.g. $u=x^2+3x\Rightarrow\int\frac{2x+3}{x^2+3x+5}\mathrm{d}x=...\ln\left(u+5\right)$
or $u=x^2+3x+5\Rightarrow\int\frac{2x+3}{x^2+3x+5}\mathrm{d}x=...\ln u$

dM1: Substitutes 20 **and** 0 into their expression of the form $\ldots\ln\left(x^2+3x+5\right)$, subtracts either way round and proceeds to find an **exact** expression for the volume e.g. $144\pi\left(\ln 465-\ln 5\right)$
If they use substitution, the limits must be **correct for their substitution** e.g.
$u=x^2+3x\Rightarrow u=0,\ 460$ or e.g. $u=x^2+3x+5\Rightarrow u=5,\ 465$
Do not allow e.g. $144\pi\left(\ln 465-0\right)$ i.e. no evidence of the use of the lower limit.
Depends on the previous mark.

A1: $144\pi\ln 93$ (condone lack of units)

Note if π is missing throughout in (a), a maximum of B0M1dM1A0 is possible.

Example working if a substitution is used e.g.

$$u=x^2+3x+5\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=2x+3\Rightarrow 144\pi\int\frac{2x+3}{u}\,\frac{\mathrm{d}x}{2x+3}=144\pi\left[\ln u\right]_5^{465}$$

$$144\pi\left(\ln 465-\ln 5\right)=144\pi\ln 93$$

(b)	$144\pi\left[\ln\left(x^2+3x+5\right)\right]_p^{20}=360\pi$ $\Rightarrow 144\pi\left(\ln 465-\ln\left(p^2+3p+5\right)\right)=360\pi$ **or e.g.** Let $p=20-a$: $144\pi\left[\ln\left(x^2+3x+5\right)\right]_{20-a}^{20}=360\pi$ $\Rightarrow 144\pi\left(\ln 465-\ln\left(\left(20-a\right)^2+3\left(20-a\right)+5\right)\right)=360\pi$ **or e.g.** $144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{20-a}=144\pi\ln 93-360\pi$ $\Rightarrow 144\pi\left(\ln\left(\left(20-a\right)^2+3\left(20-a\right)+5\right)-\ln 5\right)=144\pi\ln 93-360\pi$ **or e.g.** $144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{p}=144\pi\ln 93-360\pi$ $\Rightarrow 144\pi\left(\ln\left(p^2+3p+5\right)-\ln 5\right)=144\pi\ln 93-360\pi$	M1
	$\frac{5}{2}=\ln\left(\frac{465}{p^2+3p+5}\right)\Rightarrow e^{\frac{5}{2}}=\frac{465}{p^2+3p+5}$ $\Rightarrow p^2+3p+5-465e^{-\frac{5}{2}}=0\Rightarrow p=...$ **or e.g.** $\ln\left(\frac{a^2-43a+465}{5}\right)=\frac{144\pi\ln 93-360\pi}{144\pi}$ $\Rightarrow\frac{a^2-43a+465}{5}=e^{\left(\frac{144\ln 93-360}{144}\right)}\Rightarrow a^2-43a+465-5e^{\left(\frac{144\ln 93-360}{144}\right)}=0\Rightarrow a=...$ **or e.g.** $\ln\left(\frac{p^2+3p+5}{5}\right)=\frac{144\pi\ln 93-360\pi}{144\pi}$ $\Rightarrow\frac{p^2+3p+5}{5}=e^{\left(\frac{144\ln 93-360}{144}\right)}\Rightarrow p^2+3p+5-5e^{\left(\frac{144\ln 93-360}{144}\right)}=0\Rightarrow p=...$	**d**M1
	$a=$ awrt 15.5	A1
		(3)
		(7 marks)

(b)

M1: Sets an expression of the form $...\left[\ln\left(x^2+3x+5\right)\right]_p^{20}$ equal to 360π and substitutes in the limits 20 and p (or any other letter – condone a) and subtracts either way round.
Alternatively sets an expression of the form $...\left[\ln\left(x^2+3x+5\right)\right]_0^{20-a}$ or e.g. $...\left[\ln\left(x^2+3x+5\right)\right]_0^{p}$ (or any other letter – condone a) equal to 360π – their answer to part (a) and substitutes in the limits $20-a$ and 0 or e.g. 0 and p and subtracts either way round.
For this mark, their method, if executed correctly would either lead directly to the value of a or would enable a to be found by e.g. $a = 20 -$ their value.

dM1: **Depends on the previous mark.**
Removes lns, rearranges to form a three term quadratic and attempts to solve, proceeding to a value for p. In the alternative method this mark is scored for proceeding to find a value for