Question Number	Scheme	Marks
1	$\int 2x\ln(3x)\,\mathrm{d}x = x^2\ln(3x) - \int \frac{1}{x}\times x^2\,\mathrm{d}x$	M1
	$= ... + ...x^2$	**d**M1
	$x^2\ln(3x) - \frac{x^2}{2}$	A1
	$9\ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2}\right) = 17\ln 3 - 4$	**dd**M1A1
		(5 marks)

Mark positively in this question and do not penalise poor notation such as a missing "d*x*" or spurious integral signs, "+ *c*" etc. as long as the intention is clear.

M1: Attempts integration by parts the right way round achieving $...x^2\ln(3x) \pm ... \int \frac{1}{x}\times x^2\,\mathrm{d}x$

(You do not need to be concerned how they arrive at this)
You may see an attempt at the "DI method" e.g.

	D	I
+	$\ln(3x)$	$2x$
–	$\frac{1}{x}$	x^2

but the same condition applies e.g. they must reach $...x^2\ln(3x) \pm ... \int \frac{1}{x}\times x^2\,\mathrm{d}x$ which may be implied by e.g. $\int 2x\ln(3x)\,\mathrm{d}x = ...x^2\ln(3x) \pm ...x^2$

(In such cases, this mark and the following mark will be scored at the same time)

Condone e.g. $...\ln 3x.x^2 \pm ... \int \frac{1}{x}\times x^2\,\mathrm{d}x$

dM1: For $...x^2\ln(3x) \pm ... \int \frac{1}{x}\times x^2\,\mathrm{d}x = ... \pm ...x^2$. **Depends on the previous method mark.**

A1: Fully correct integration: $x^2\ln(3x) - \frac{x^2}{2}$ or unsimplified equivalent e.g. $2\left\{\frac{1}{2}x^2\ln(3x) - \frac{x^2}{4}\right\}$ with or without a constant of integration.

Any necessary brackets must be present unless they are implied by subsequent work.

Condone e.g. $\ln(3x).x^2 - \frac{x^2}{2}$ but not e.g. $\ln(3x^3) - \frac{x^2}{2}$ unless $\ln(3x).x^2 - \frac{x^2}{2}$ is seen first.

ddM1: Substitutes in the limits 3 **and** 1 into an expression of the form $\alpha x^2\ln(3x) + \beta x^2$ and subtracts the correct way round. Condone missing brackets e.g. $9\ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[x^2\ln(3x) - \frac{x^2}{2}\right]_1^3 = \left[x^2\ln(3x)\right]_1^3 - \left[\frac{x^2}{2}\right]_1^3 = 9\ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2}\right)$$

It is dependent on the two previous method marks.

A1: $17\ln 3 - 4$ Following correct work. Condone $17\ln 3 + -4$

Note: A common slip is to differentiate $\ln(3x)$ to obtain $\frac{1}{3x}$ leading to :

$$\int 2x\ln(3x)\,dx = x^2\ln(3x) - \int \frac{1}{3x}\times x^2\,dx$$

$$= x^2\ln(3x) - \frac{x^2}{6} \rightarrow \left[x^2\ln(3x) - \frac{x^2}{6}\right]_1^3 = 9\ln 9 - \frac{9}{6} - \left(\ln 3 - \frac{1}{6}\right) = 17\ln 3 - \frac{4}{3}$$

and this scores a maximum M1dM1A0ddM1A0

Note: A common incorrect approach is to integrate in the wrong way round with various incorrect attempts to integrate $\ln(3x)$ e.g.

$$\int 2x\ln(3x)\,dx = \frac{2x}{3x} - \int \frac{2}{3x}dx \text{ etc.}$$

This generally scores no marks.

	Alternative by substitution $u=3x$	
	e.g. $u=3x \Rightarrow \int 2x\ln(3x)\,\mathrm{d}x = \int \frac{2}{3}u\ln u \times \frac{1}{3}\mathrm{d}u$ $= \frac{1}{9}u^2\ln u - \frac{2}{9}\int \frac{u^2}{2}\times\frac{1}{u}\mathrm{d}u$	M1
	$= ... + ...u^2$	**d**M1
	$\frac{1}{9}u^2\ln u - \frac{2}{9}\times\frac{u^2}{4}$	A1
	$x=1,\ 3 \Rightarrow u=3,\ 9$ $9\ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2}\right) = 17\ln 3 - 4$	**dd**M1A1

M1: Attempts integration by parts the right way round achieving $...u^2\ln(u) \pm ... \int \frac{1}{u}\times u^2\,\mathrm{d}u$

Condone e.g. $= \frac{1}{9}\ln u.u^2 - \frac{2}{9}\int \frac{u^2}{2}\times\frac{1}{u}\mathrm{d}u$

dM1: For $...u^2\ln(u) \pm ... \int \frac{1}{u}\times u^2\,\mathrm{d}u ... \pm ...u^2$. **Depends on the previous method mark.**

Condone e.g. $\frac{1}{9}\ln u.u^2 - \frac{2}{9}\times\frac{u^2}{4}$

A1: Fully correct integration for their substitution: $\frac{1}{9}u^2\ln u - \frac{2}{9}\times\frac{u^2}{4}$ or unsimplified equivalent e.g.

$\frac{2}{9}\left\{\frac{1}{2}u^2\ln u - \frac{u^2}{4}\right\}$ with or without a constant of integration.

Condone e.g. $\frac{1}{9}\ln u.u^2 - \frac{2}{9}\times\frac{u^2}{4}$ but not e.g. $\frac{1}{9}\ln u^3 - \frac{2}{9}\times\frac{u^2}{4}$ unless $\frac{1}{9}\ln u.u^2 - \frac{2}{9}\times\frac{u^2}{4}$ is seen first.

Any necessary brackets must be present unless they are implied by subsequent work.

ddM1: Substitutes in the limits 9 **and** 3 into an expression of the form $\alpha u^2\ln(u) + \beta u^2$ or reverts back to x and substitutes in the limits 3 **and** 1 and subtracts the correct way round.

Condone missing brackets e.g. $9\ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[\frac{1}{9}u^2\ln u - \frac{u^2}{18}\right]_3^9 = \left[\frac{1}{9}u^2\ln u\right]_3^9 - \left[\frac{u^2}{18}\right]_3^9 = 9\ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2}\right)$$

It is dependent on the two previous method marks.

A1: $17\ln 3 - 4$

	Alternative by substitution $u=\ln(3x)$**.**	
	e.g. $u=\ln(3x)\Rightarrow\int 2x\ln(3x)\,\mathrm{d}x=\int\frac{2}{3}\mathrm{e}^{u}u\times\frac{1}{3}\mathrm{e}^{u}\mathrm{d}u=\frac{2}{9}\int u\mathrm{e}^{2u}\mathrm{d}u$ $=\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{9}\int\mathrm{e}^{2u}\mathrm{d}u$	M1
	$=\ldots+\ldots\mathrm{e}^{2u}$	**d**M1
	$\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{18}\mathrm{e}^{2u}$	A1
	$x=1,\ 3\Rightarrow u=\ln 3,\ \ln 9$ $9\ln 9-\frac{9}{2}-\left(\ln 3-\frac{1}{2}\right)=17\ln 3-4$	**dd**M1A1

M1: Attempts integration by parts the right way round achieving $\ldots u\mathrm{e}^{2u}\pm\ldots\int\mathrm{e}^{2u}\mathrm{d}u$

dM1: For $\ldots u\mathrm{e}^{2u}\pm\ldots\int\mathrm{e}^{2u}\mathrm{d}u=\ldots\pm\ldots\mathrm{e}^{2u}$. **Depends on the previous method mark.**

A1: Fully correct integration for their substitution: $\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{18}\mathrm{e}^{2u}$ or unsimplified equivalent e.g. $\frac{2}{9}\left(\frac{1}{2}u\mathrm{e}^{2u}-\frac{1}{4}\mathrm{e}^{2u}\right)$ with or without a constant of integration.

Any necessary brackets must be present unless they are implied by subsequent work.

ddM1: Substitutes in the limits ln 9 **and** ln 3 into an expression of the form $\alpha u\mathrm{e}^{2u}+\beta\mathrm{e}^{2u}$ or reverts back to x and substitutes in the limits 3 **and** 1 and subtracts the correct way round.

Condone missing brackets e.g. $9\ln 9-\frac{9}{2}-\ln 3-\frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[\frac{1}{9}u\mathrm{e}^{2u}-\frac{1}{18}\mathrm{e}^{2u}\right]_{\ln 3}^{\ln 9}=\left[\frac{1}{9}u\mathrm{e}^{2u}\right]_{\ln 3}^{\ln 9}-\left[\frac{1}{18}\mathrm{e}^{2u}\right]_{\ln 3}^{\ln 9}=9\ln 9-\ln 3-\left(\frac{9}{2}-\frac{1}{2}\right)$$

It is dependent on the two previous method marks.

A1: $17\ln 3-4$

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx} = 3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3x^2}$	**d**M1A1
		(3)
	Alternative 1	
	$V = x^3 \Rightarrow x = V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV} = \frac{1}{3}V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3V^{\frac{2}{3}}} = \frac{5}{3x^2}$	**d**M1A1
	Alternative 2	
	$\frac{dV}{dt} = 5 \Rightarrow V = 5t + c$	M1 (B1 on EPEN)
	$V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 \frac{dx}{dt} = 5$ or $V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 = 5\frac{dt}{dx}$	**d**M1
	$\frac{dx}{dt} = \frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt} = \frac{5}{3 \times 4^2} = \frac{5}{48}$	M1A1
		(2)
		(5 marks)

Note this is now being marked as M1dM1A1 not B1M1A1

(a) **Ignore any units associated with any of the expressions/values in this question.**

M1: Differentiates $V = x^3$ to obtain $\frac{dV}{dx} = ...x^2$ (seen or implied)

dM1: Attempts to use $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ with their $\frac{dV}{dx}$ and $\frac{dV}{dt}$ where $\frac{dV}{dx} = kx^2$ and $\frac{dV}{dt}$ is a constant to obtain an expression of the form $\frac{\alpha}{x^2}$ oe.

May be implied by their working.

Depends on the previous mark.

A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.