(b)	$144\pi\left[\ln\left(x^2+3x+5\right)\right]_p^{20}=360\pi$ $\Rightarrow 144\pi\left(\ln 465-\ln\left(p^2+3p+5\right)\right)=360\pi$ **or e.g.** Let $p=20-a$: $144\pi\left[\ln\left(x^2+3x+5\right)\right]_{20-a}^{20}=360\pi$ $\Rightarrow 144\pi\left(\ln 465-\ln\left(\left(20-a\right)^2+3\left(20-a\right)+5\right)\right)=360\pi$ **or e.g.** $144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{20-a}=144\pi\ln 93-360\pi$ $\Rightarrow 144\pi\left(\ln\left(\left(20-a\right)^2+3\left(20-a\right)+5\right)-\ln 5\right)=144\pi\ln 93-360\pi$ **or e.g.** $144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^{p}=144\pi\ln 93-360\pi$ $\Rightarrow 144\pi\left(\ln\left(p^2+3p+5\right)-\ln 5\right)=144\pi\ln 93-360\pi$	M1
	$\frac{5}{2}=\ln\left(\frac{465}{p^2+3p+5}\right)\Rightarrow e^{\frac{5}{2}}=\frac{465}{p^2+3p+5}$ $\Rightarrow p^2+3p+5-465e^{-\frac{5}{2}}=0\Rightarrow p=...$ **or e.g.** $\ln\left(\frac{a^2-43a+465}{5}\right)=\frac{144\pi\ln 93-360\pi}{144\pi}$ $\Rightarrow\frac{a^2-43a+465}{5}=e^{\left(\frac{144\ln 93-360}{144}\right)}\Rightarrow a^2-43a+465-5e^{\left(\frac{144\ln 93-360}{144}\right)}=0\Rightarrow a=...$ **or e.g.** $\ln\left(\frac{p^2+3p+5}{5}\right)=\frac{144\pi\ln 93-360\pi}{144\pi}$ $\Rightarrow\frac{p^2+3p+5}{5}=e^{\left(\frac{144\ln 93-360}{144}\right)}\Rightarrow p^2+3p+5-5e^{\left(\frac{144\ln 93-360}{144}\right)}=0\Rightarrow p=...$	**d**M1
	$a=$ awrt 15.5	A1
		(3)
		(7 marks)

(b)

M1: Sets an expression of the form $...\left[\ln\left(x^2+3x+5\right)\right]_p^{20}$ equal to 360π and substitutes in the limits 20 and p (or any other letter – condone a) and subtracts either way round.
Alternatively sets an expression of the form $...\left[\ln\left(x^2+3x+5\right)\right]_0^{20-a}$ or e.g. $...\left[\ln\left(x^2+3x+5\right)\right]_0^{p}$ (or any other letter – condone a) equal to 360π – their answer to part (a) and substitutes in the limits $20-a$ and 0 or e.g. 0 and p and subtracts either way round.
For this mark, their method, if executed correctly would either lead directly to the value of a or would enable a to be found by e.g. $a = 20 -$ their value.

dM1: **Depends on the previous mark.**
Removes lns, rearranges to form a three term quadratic and attempts to solve, proceeding to a value for p. In the alternative method this mark is scored for proceeding to find a value for

	a. The usual rules apply for solving their 3TQ although this is most likely to be done on a calculator so you may need to check.
A1:	cao awrt 15.5 with or without units Note that if they find "*p*" first then then need to find 20 – "*p*" to obtain the correct value of *a*. For reference, "*p*" = 4.45143…..

Examples in (b) that score no marks:

$$144\pi\left[\ln\left(x^2+3x+5\right)\right]_0^p = 360\pi$$

$$144\pi\left[\ln u\right]_5^{p^2+3p+5} = 360\pi$$

(Having substituted $u = x^2+3x+5$ in (a))

Example of correct working if a substitution is used e.g. $u = x^2+3x+5$

$$\text{Let } p = 20-a: \quad 144\pi\left[\ln u\right]_{p^2+3p+5}^{465} = 360\pi$$

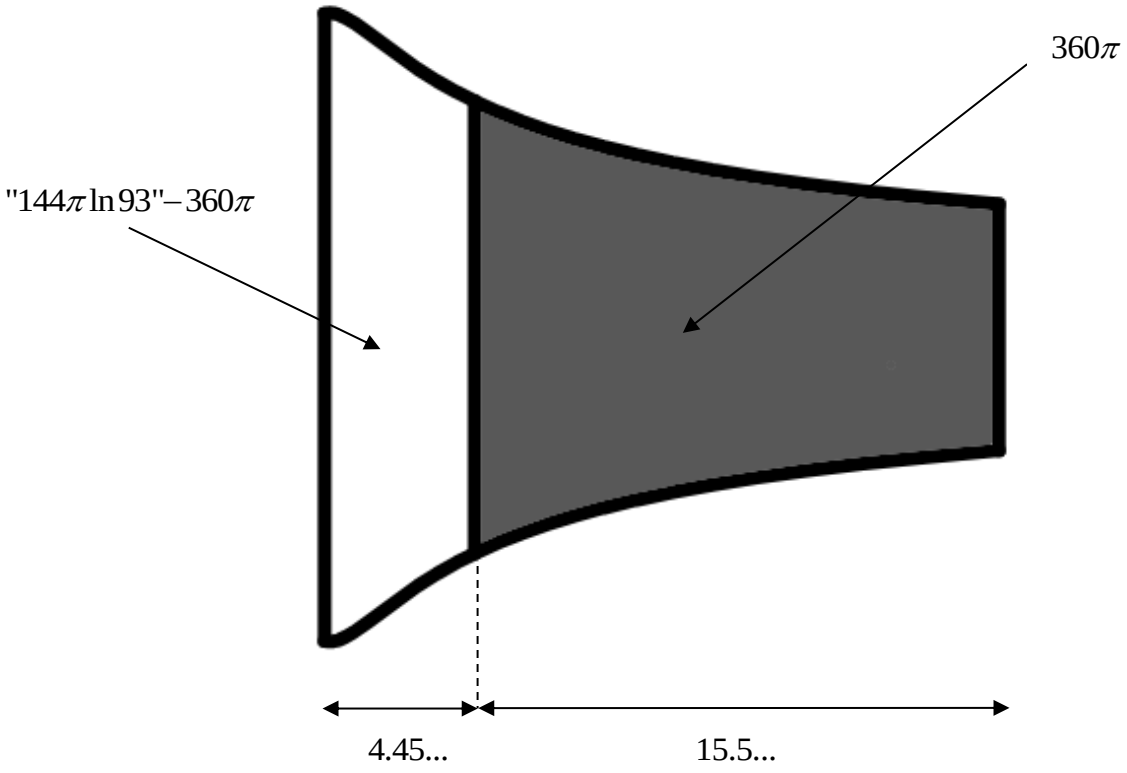
$$\Rightarrow \frac{5}{2} = \ln\left(\frac{465}{p^2+3p+5}\right) \Rightarrow e^{\frac{5}{2}} = \frac{465}{p^2+3p+5} \text{ etc.}$$

or e.g.

$$144\pi\left[\ln u\right]_5^{(20-a)^2+3(20-a)+5} = 144\pi\ln 93 - 360\pi$$

$$\Rightarrow 144\pi\left(\ln\left((20-a)^2+3(20-a)+5\right)-\ln 5\right) = 144\pi\ln 93 - 360\pi \text{ etc.}$$

<u>For reference:</u>



Question 10

Note that there are many different ways to answer this question.

General Guidance:

B1: For setting up the correct contradiction.
It must include a word/words such as "assume" or "let" or "if" and $p^2 - 4q + 2 = 0$
As a minimum accept just "assume/let/if $p^2 - 4q + 2 = 0$"

M1: Attempts to make p^2 the subject.

A1: Makes a correct deduction following correct work.

dM1: Continues the argument/algebra that will lead to a contradiction.

A1: For a fully correct proof.
Requires

- all previous marks scored
- no incorrect assumptions/statements
- correct calculations/algebra
- a reason: e.g. $2q - 2k^2 = 1$ is not true
- E.g. hence $p^2 - 4q + 2 \neq 0$, hence proven, QED, etc. but not just "contradiction"

Question Number	Scheme	Marks
10 Main	Assume that $p^2-4q+2=0$	B1
	$p^2=4q-2$	M1
	$p^2=4q-2\left(=2(2q-1)\right)$ So p^2 is even so p is even oe e.g. so $p=2k$	A1
	If $p=2k$ then e.g. $4k^2=4q-2$ or e.g. $4k^2=4q-2\Rightarrow q=k^2+\frac{1}{2}$ or e.g. $q=\frac{2k^2+1}{2}$ or e.g. $4k^2=4q-2\Rightarrow k^2=q-\frac{1}{2}$ or e.g. $k^2=\frac{2q-1}{2}$ or e.g. Let $4k^2=4q-2\Rightarrow 2q-2k^2=1$	**d**M1
	$4k^2$ is a multiple of 4 and $4q-2$ isn't a multiple of 4 or e.g. $q=k^2+\frac{1}{2}$ which is not an integer or e.g. $k^2=q-\frac{1}{2}$ which is not an integer or e.g. $2q-2k^2=1$ which is not possible So $p^2-4q+2\neq 0$	A1
		(5 marks)

Way 1

B1: For setting up the correct contradiction.
It must include a word/words such as "assume" or "let" or "if" **and** $p^2-4q+2=0$
As a minimum accept just "assume/let/if" $p^2-4q+2=0$"

M1: Rearranges $p^2-4q+2=0$ to obtain $p^2=\pm 4q\pm 2$

A1: Correct deduction for *p* following **correct work**.
It is not necessary to factorise $4q-2$ but if they do it must be correct.
It requires e.g. both p^2 is even **and** so *p* is even oe e.g. so $p=2k$ or any other letter but not $p = 2q$ or $p = 2p$

dM1: **Depends on the previous method mark.**
For attempting to use the fact that *p* is even to obtain a contradiction e.g. uses that as *p* is even $p = 2k$ and substitutes into the equation to obtain e.g.

$4k^2=4q-2$ or $q=k^2+\frac{1}{2}$ or $k^2=q-\frac{1}{2}$ or $2q-2k^2=1$

A1: Fully correct work with all previous marks scored that includes

- An explanation why there is a contradiction
- A (minimal) conclusion e.g. "so $p^2-4q+2\neq 0$", QED, "proven", ✓

Must follow all previous marks and there must be no incorrect statements/algebra in their proof.