8. (a) Find, in ascending powers of x, the first three non-zero terms of the binomial series expansion of

$$\frac{1}{\sqrt{4+x}}$$

giving each coefficient as a simplified fraction.

(4)

Using the expansion from part (a),

(b) state the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{4-x}}$

(1)

(c) Hence, or otherwise, show that

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} \approx a + bx^2$$

where a and b are fully simplified fractions to be found.

(2)

(d) Use

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} = a + bx^2$$

with $x = 1$ and the values of a and b to find a fully simplified rational approximation for $\sqrt{135}$

Show your working and make your method clear.

(3)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

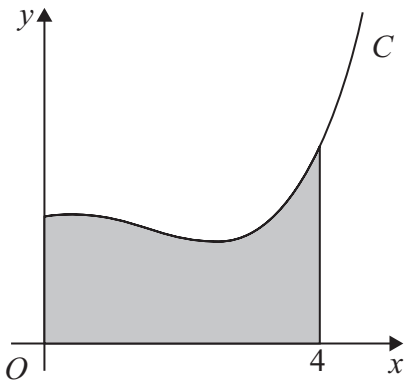


Figure 1

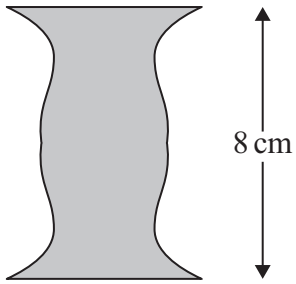


Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^{x}$$

The finite region, shown shaded in Figure 1, is bounded by C, the y-axis, the x-axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x-axis to form a solid S.

(a) Show that the exact volume of S is given by

$$\pi \int_0^4 \left(A + B\cos 2x + Ce^{x}\cos x + De^{2x} \right) dx$$

where A, B, C and D are constants to be found.

(4)

(b) Find $\int e^{x}\cos x \, dx$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together.
The paperweight is 8 cm high.

(c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)

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