Question Number	Scheme	Marks
7(a)	$u = \tan x \rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$	B1
	$u^2 = \tan^2 x = \sec^2 x - 1 \rightarrow \int \dfrac{u+u^3}{(5+u^2)^3} \times \dfrac{1}{1+u^2}\ \mathrm{d}u$	M1dM1
	$\int \dfrac{u}{(5+u^2)^3}\ \mathrm{d}u$ *	A1*
		(4)
(b)	$\int \dfrac{u}{(5+u^2)^3}\ \mathrm{d}u = -\dfrac{1}{4}(5+u^2)^{-2} + c = -\dfrac{1}{4}(5+\tan^2 x)^{-2}\,(+c)$	M1A1
		(2)
		(6 marks)

(a)

B1: $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$ or exact equivalent seen or used. For example, $\dfrac{\mathrm{d}x}{\mathrm{d}u} = \dfrac{1}{1+u^2}$ is correct

M1: Attempts to use $\pm 1 \pm \tan^2 x = \sec^2 x$ in a correct place at least once. Implied by use of $\sec^2 x = \pm 1 \pm u^2$

dM1: Fully substitutes $u = \tan x$ proceeding to an integral in terms of <u>u only.</u>
It is dependent upon the previous M

Look for $\left(\int \dfrac{\tan x + \tan^3 x}{(4+\sec^2 x)^3}\ \mathrm{d}x\right) \rightarrow \int \dfrac{u+u^3}{(4 \pm 1 \pm u^2)^3} \times \mathrm{f}(u)\ \mathrm{d}u$ seen on at least one line

Condone a lack of an integral sign. You may never see the integral in terms of *x*. (It is in the question)

A1*: Proceeds with no errors in the body of their proof to $\int \dfrac{u}{(5+u^2)^3}\ \mathrm{d}u$ including the d*u* .

The d*x*'s and d*u*'s must be present and correct for their stage of working throughout the proof.

(b)

M1: Attempts to integrate an expression of the form $\int \dfrac{u}{(k+u^2)^n}\ \mathrm{d}u \rightarrow A(k+u^2)^{-n+1}$ where *A* is a constant and $n \neq 1$, $n \in \mathrm{N}$

A1: $-\dfrac{1}{4}(5+\tan^2 x)^{-2} + c$ or equivalent e.g. $-\dfrac{1}{4}(4+\sec^2 x)^{-2} + c$. (condone omission of +*c*)

This must be in terms of *x* and you can ISW after sight of a correct answer.

Question Number	Scheme	Marks
8(a)	$\frac{1}{\sqrt{4+x}} = 4^{-\frac{1}{2}}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$	M1
	$\left(1+\frac{x}{4}\right)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\right)\left(\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{x}{4}\right)^2+\ldots$	M1
	$\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2+\ldots$	A1A1
		(4)
(b)	$\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2+\ldots$	B1ft
		(1)
(c)	$\left(\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2\right)\left(\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2\right)=\frac{1}{4}+\frac{1}{128}x^2$	M1A1
		(2)
(d)	$\frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{3}}="\frac{1}{4}"+"\frac{1}{128}"="\frac{33}{128}"$	M1
	$"\frac{128}{33}"\times 3=\frac{128}{11}$ (see alt (d) for alternative answer)	dM1A1
		(3)
Alt(d)	$\frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{3}}="\frac{1}{4}"+"\frac{1}{128}"="\frac{33}{128}"$	M1
	$\frac{1}{\sqrt{15}}\times\frac{\sqrt{15}}{\sqrt{15}}=\frac{\sqrt{15}}{15}\approx"\frac{33}{128}"\Rightarrow\sqrt{135}=3\sqrt{15}\approx 3\times 15\times"\frac{33}{128}"=\frac{1485}{128}$	dM1A1
		(10 marks)

(a) It is scored B, M, A, A on epen. We are scoring it M, M, A, A

M1: Takes out a factor of 4 proceeding to $4^{-\frac{1}{2}}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ o.e such as $\frac{1}{2}\times\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$

M1: Attempts the binomial expansion.

Look for the correct binomial coefficient for the **third term with the correct power of** *x*. So, award for $\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(\ldots x)^2$ or $\underline{\underline{\frac{3}{8}}}\times\ldots\underline{\underline{x^2}}$ where … could even be 1. If there is no intermediate working it is implied by the correct coefficient of $\frac{3}{128}x^2$ in the expansion of $\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ or $\frac{3}{256}x^2$ in the expansion of $(4+x)^{-\frac{1}{2}}$

A1: Two correct and simplified of $\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2+\ldots$ So 1,0,1, 0 is possible

A1: All three correct and simplified of $\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2+\ldots$