7. (a) Show that the substitution $u = \tan x$ transforms

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} \, \mathrm{d}x$$

to

$$\int \frac{u}{(k + u^2)^n} \, \mathrm{d}u$$

where k and n are integers to be found.

(4)

(b) Hence find

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} \, \mathrm{d}x$$

(2)

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Question 7 continued

(Total for Question 7 is 6 marks)

8. (a) Find, in ascending powers of x, the first three non-zero terms of the binomial series expansion of

$$\frac{1}{\sqrt{4+x}}$$

giving each coefficient as a simplified fraction.

(4)

Using the expansion from part (a),

(b) state the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{4-x}}$

(1)

(c) Hence, or otherwise, show that

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} \approx a + bx^2$$

where a and b are fully simplified fractions to be found.

(2)

(d) Use

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} = a + bx^2$$

with $x = 1$ and the values of a and b to find a fully simplified rational approximation for $\sqrt{135}$

Show your working and make your method clear.

(3)

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