Question Number	Scheme	Marks
6(a)	Attempts $\pm\left(\left(5\mathbf{i}+2\mathbf{j}+\mathbf{k}\right)-\left(\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)\right)=\pm\left(4\mathbf{i}+4\mathbf{k}\right)$	M1
	e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$	B1
	$\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{4^2+4^2}$ or $\left\lvert\overrightarrow{AC}\right\rvert=\sqrt{2^2+(\alpha-2)^2+8^2}$	M1
	$\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{32}$ and $\left\lvert\overrightarrow{AC}\right\rvert=\sqrt{68+(\alpha-2)^2}$	A1
	$\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$	dM1
	$\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$	ddM1
	$\alpha=2\pm4\sqrt{2}$	A1
		(6)
		(8 marks)

Allow vectors to be written in any acceptable form. E.g. column vectors are fine

(a)

M1: Attempts to find the difference between the two vectors.
If no method is shown look for two correct components from $\pm(4\mathbf{i}),(0\mathbf{j}),\pm(4\mathbf{k})$

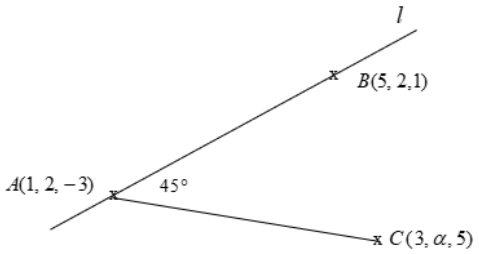
A1: Any correct equation for the line in the form given.
Examples of correct equations are;
$\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$, $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+1\mathbf{k}+\mu(\mathbf{i}+\mathbf{k})$

$\mathbf{r}=\begin{pmatrix}5\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}-4\\0\\-4\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}2\\2\\-2\end{pmatrix}+k\begin{pmatrix}2\\0\\2\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}3\\2\\-1\end{pmatrix}+\delta\begin{pmatrix}-1\\0\\-1\end{pmatrix}$

Note that the LHS must be correct so $l=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ is A0

Condone the vectors not being bold or underlined

Condone incorrect vector notation for the M but not the A mark. E.g. $\mathbf{r}=\begin{pmatrix}5\mathbf{i}\\2\mathbf{j}\\\mathbf{k}\end{pmatrix}+\lambda\begin{pmatrix}-4\mathbf{i}\\0\\-4\mathbf{k}\end{pmatrix}$ is M1A0



(b) **There are many ways to attempt this question.**

The following is via the scalar product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Note that it could be attempted via the scalar product of $\pm(1\mathbf{i}+1\mathbf{k})$ and $\overrightarrow{AC}$ instead of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Attempts using an incorrect vector pair, e.g $\overrightarrow{AB}$ and $\overrightarrow{BC}$, can only score a maximum of 1 mark (1st M1)

Condone incorrect vector notation throughout (b). E.g. $\overrightarrow{AC}=(2\mathbf{i},(\alpha-2)\mathbf{j},8\mathbf{k})$

B1: A correct $\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$ (or $\overrightarrow{CA}=-2\mathbf{i}-(\alpha-2)\mathbf{j}-8\mathbf{k}$)

It must be simplified but may be implied by further work.

Allow in coordinate form $(2,\alpha-2,8)$ if used correctly

M1: Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$

For example, for an attempt at the magnitude of $\overrightarrow{AC}$

- there should be attempt to subtract each component of $\overrightarrow{OA}$ and $\overrightarrow{OC}$
- the three resulting values should then be squared and added
- the answer to this is then square rooted

If the method is not apparent then look for two correct components

A1: Correct magnitude of both $\left|\overrightarrow{AB}\right|=\sqrt{32}$ and $\left|\overrightarrow{AC}\right|=\sqrt{68+(\alpha-2)^2}$ $\left(=\sqrt{72-4\alpha+a^2}\right)$

They must be simplified (as above) but can be scored following sign slips on either vector.

You can imply this mark from further work

dM1: Attempts the scalar product of $\pm'\overrightarrow{AB}'$ and $\pm'\overrightarrow{AC}'$

- $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$ must be attempted via a correct method. If the method is not directly seen it may be implied by two correct components
- There must be a similar correct attempt at the magnitudes of $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$
- All should be connected by the equation $\pm\overrightarrow{AB}\bullet\pm\overrightarrow{AC}=\left|\pm\overrightarrow{AB}\right|\left|\pm\overrightarrow{AC}\right|\cos 45$

ddM1: Allowable method to find at least one value of α from a quadratic equation.

For example,

- $(\alpha-2)^2=32\Rightarrow\alpha=...$
- $\alpha^2-4\alpha-28=0\Rightarrow\alpha=...$

This can only be awarded if the equation has real roots.

It is dependent upon both previous M's

A1: $\alpha=2\pm4\sqrt{2}$ or exact equivalent

Note: Alternative method via cosine rule

B1: $\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$ (or $\overrightarrow{CA}=-2\mathbf{i}-(\alpha-2)\mathbf{j}-8\mathbf{k}$) May be implied by further work.

M1: Attempts to find the magnitude of either $\overrightarrow{AB}$ or $\overrightarrow{AC}$

A1: $\left|\overrightarrow{AB}\right|=\sqrt{32}$ and $\left|\overrightarrow{AC}\right|=\sqrt{68+(\alpha-2)^2}$ $\left(=\sqrt{72-4\alpha+a^2}\right)$

dM1: Attempts $\left|BC\right|^2=\left|AB\right|^2+\left|AC\right|^2-2\left|AB\right|\left|AC\right|\cos 45$

FYI $\left(2^2+(\alpha-2)^2+4^2\right)=\left(2^2+(\alpha-2)^2+8^2\right)+\left(4^2+4^2\right)-2\sqrt{\left(2^2+(\alpha-2)^2+4^2\right)}\sqrt{\left(4^2+4^2\right)}\times\frac{\sqrt{2}}{2}$

ddM1: Rearranges and proceeds to at least one value for α. This can only be awarded if the equation is quadratic and has real roots

A1: $\alpha=2\pm4\sqrt{2}$ or exact equivalent

Question Number	Scheme	Marks
7(a)	$u = \tan x \rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$	B1
	$u^2 = \tan^2 x = \sec^2 x - 1 \rightarrow \int \dfrac{u+u^3}{(5+u^2)^3} \times \dfrac{1}{1+u^2}\ \mathrm{d}u$	M1dM1
	$\int \dfrac{u}{(5+u^2)^3}\ \mathrm{d}u$ *	A1*
		(4)
(b)	$\int \dfrac{u}{(5+u^2)^3}\ \mathrm{d}u = -\dfrac{1}{4}(5+u^2)^{-2} + c = -\dfrac{1}{4}(5+\tan^2 x)^{-2}\,(+c)$	M1A1
		(2)
		(6 marks)

(a)

B1: $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$ or exact equivalent seen or used. For example, $\dfrac{\mathrm{d}x}{\mathrm{d}u} = \dfrac{1}{1+u^2}$ is correct

M1: Attempts to use $\pm 1 \pm \tan^2 x = \sec^2 x$ in a correct place at least once. Implied by use of $\sec^2 x = \pm 1 \pm u^2$

dM1: Fully substitutes $u = \tan x$ proceeding to an integral in terms of <u>u only.</u>
It is dependent upon the previous M

Look for $\left(\int \dfrac{\tan x + \tan^3 x}{(4+\sec^2 x)^3}\ \mathrm{d}x\right) \rightarrow \int \dfrac{u+u^3}{(4 \pm 1 \pm u^2)^3} \times \mathrm{f}(u)\ \mathrm{d}u$ seen on at least one line

Condone a lack of an integral sign. You may never see the integral in terms of *x*. (It is in the question)

A1*: Proceeds with no errors in the body of their proof to $\int \dfrac{u}{(5+u^2)^3}\ \mathrm{d}u$ including the d*u* .

The d*x*'s and d*u*'s must be present and correct for their stage of working throughout the proof.

(b)

M1: Attempts to integrate an expression of the form $\int \dfrac{u}{(k+u^2)^n}\ \mathrm{d}u \rightarrow A(k+u^2)^{-n+1}$ where *A* is a constant and $n \neq 1$, $n \in \mathrm{N}$

A1: $-\dfrac{1}{4}(5+\tan^2 x)^{-2} + c$ or equivalent e.g. $-\dfrac{1}{4}(4+\sec^2 x)^{-2} + c$. (condone omission of +*c*)

This must be in terms of *x* and you can ISW after sight of a correct answer.