Question Number	Scheme	Marks
9(a)	$(V)=\pi\int_0^4\left(\cos x+\frac{1}{5}\mathrm{e}^x\right)^2\,\mathrm{d}x$	B1
	$\left(\cos x+\frac{1}{5}\mathrm{e}^x\right)^2=\cos^2 x+\frac{2}{5}\mathrm{e}^x\cos x+\frac{1}{25}\mathrm{e}^{2x}$	M1
	$=\frac{\cos 2x+1}{2}+\frac{2}{5}\mathrm{e}^x\cos x+\frac{1}{25}\mathrm{e}^{2x}$	dM1
	$=\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}\mathrm{e}^x\cos x+\frac{1}{25}\mathrm{e}^{2x}\right)\mathrm{d}x$	A1
		(4)
(b)	**Way I** $\int \mathrm{e}^x\cos x\,\mathrm{d}x=\pm\mathrm{e}^x\sin x\pm\int\mathrm{e}^x\sin x\,\mathrm{d}x$ $=\mathrm{e}^x\sin x-\int\mathrm{e}^x\sin x\,\mathrm{d}x$ **Way Two** $\int \mathrm{e}^x\cos x\,\mathrm{d}x=\mathrm{e}^x\cos x\pm\int\mathrm{e}^x\sin x\,\mathrm{d}x$ $=\mathrm{e}^x\cos x+\int\mathrm{e}^x\sin x\,\mathrm{d}x$	M1 A1
	$\int \mathrm{e}^x\cos x\,\mathrm{d}x=\mathrm{e}^x\cos x\pm\mathrm{e}^x\sin x-\int\mathrm{e}^x\cos x\,\mathrm{d}x$ $2\int \mathrm{e}^x\cos x\,\mathrm{d}x=\mathrm{e}^x\cos x\pm\mathrm{e}^x\sin x$	dM1
	$\int \mathrm{e}^x\cos x\,\mathrm{d}x=\frac{1}{2}\mathrm{e}^x\left(\cos x+\sin x\right)+(c)$	A1
		(4)
(c)	$\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}\mathrm{e}^x\cos x+\frac{1}{25}\mathrm{e}^{2x}\right)\mathrm{d}x$ $=(\pi)\left[\frac{1}{2}x+\frac{\sin 2x}{4}+\frac{2}{5}\left(\frac{1}{2}\mathrm{e}^x\left(\sin x+\cos x\right)\right)+\frac{1}{50}\mathrm{e}^{2x}\right]_0^4$	M1A1ft
	$2\pi\left(2+\frac{\sin 8}{4}+\frac{2}{5}\left(\frac{1}{2}\mathrm{e}^4\left(\sin(4)+\cos(4)\right)\right)+\frac{1}{50}\mathrm{e}^8-\frac{11}{50}\right)$	dM1
	$=\text{awrt } 290\ (\text{cm}^3)$	A1
		(4)
		(12 marks)

(a)

B1: A correct expression for the volume including limits and dx E.g. $\pi\int_0^4\left(\cos x+\frac{1}{5}\mathrm{e}^x\right)^2\mathrm{d}x$

The correct limits may be implied by subsequent work. It can be awarded from a correct final answer (as it is not a given answer). Don't be concerned by the non- appearance of a V. Condone it being termed 'Area'

M1: Attempts to multiply out $\left(\cos x+\frac{1}{5}\mathrm{e}^x\right)^2$ leading to $\cos^2 x\pm...\mathrm{e}^x\cos x\pm...\mathrm{e}^{2x}$

dM1: Uses the identity $\cos 2x=\pm 2\cos^2 x\pm 1$. Dependent upon previous M

A1: Fully correct $\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}\mathrm{e}^x\cos x+\frac{1}{25}\mathrm{e}^{2x}\right)\mathrm{d}x$

For this final mark

- condone either a missing dx or limits (but not both) if the candidate has already lost the B mark for the same reason
- condone missing limits if they have already appeared within an expression for the volume anywhere in part (a)

(b) Condone lack of dx's throughout part (b).
If you see other methods that you feel deserve credit, and you cannot apply the scheme, then please send to review.

M1: Attempts integration by parts. Be generous unless they explicitly state the rule is $\int v\,\mathrm{d}u = uv + \int u\,\mathrm{d}v$

Score for Way One $\int \mathrm{e}^x \cos x\ \mathrm{d}x = \pm\mathrm{e}^x \sin x \pm \int \mathrm{e}^x \sin x\ \mathrm{d}x$

Or Way Two $\int \mathrm{e}^x \cos x\ \mathrm{d}x = \mathrm{e}^x \cos x \pm \int \mathrm{e}^x \sin x\ \mathrm{d}x$

A1: Correct intermediate form

Score for Way One $\int \mathrm{e}^x \cos x\ \mathrm{d}x = \mathrm{e}^x \sin x - \int \mathrm{e}^x \sin x\ \mathrm{d}x$

Or Way Two $\int \mathrm{e}^x \cos x\ \mathrm{d}x = \mathrm{e}^x \cos x + \int \mathrm{e}^x \sin x\ \mathrm{d}x$

dM1: Attempts integration by parts a second time. It is dependent upon having scored the previous M.

Score for $\int \mathrm{e}^x \cos x\ \mathrm{d}x = \mathrm{e}^x \cos x \pm \mathrm{e}^x \sin x - \int \mathrm{e}^x \cos x\ \mathrm{d}x$

AND rearranges to $2\int \mathrm{e}^x \cos x\ \mathrm{d}x = \mathrm{e}^x \cos x \pm \mathrm{e}^x \sin x$

A1: CSO $\int \mathrm{e}^x \cos x\ \mathrm{d}x = \frac{1}{2}\mathrm{e}^x\left(\cos x + \sin x\right) + c$ o.e. condone lack of +c and the dx

Withold this mark if there is incorrect or unexplained working within the proof

(c)

M1: Attempts to integrate their (a) condoning transcription errors on coefficients achieving $\alpha x + \beta \sin 2x + \chi\left("\frac{1}{2}\mathrm{e}^x\left(\cos x + \sin x\right)"\right) + \delta\mathrm{e}^{2x}$ where α, β, χ and δ are constants following through on their attempted integration of $\mathrm{e}^x \cos x$

They must have an attempted integration for part (b), which may be incorrect, which is then used in (c).

A1ft: $(\pi)\left(\frac{1}{2}x + \frac{\sin 2x}{4} + \frac{2}{5}\left(\frac{1}{2}\mathrm{e}^x\left(\cos x + \sin x\right)\right) + \frac{1}{50}\mathrm{e}^{2x}\right)$

Condone the absence of π and follow through their coefficients A, B, C and D from (a). The integration from part (c) must be correct.

So, look for $\int_0^4 \left(A + B\cos 2x + C\mathrm{e}^x \cos x + D\mathrm{e}^{2x}\right)\mathrm{d}x \to \left(Ax + \frac{B}{2}\sin 2x + C\left(\frac{1}{2}\mathrm{e}^x\left(\cos x + \sin x\right)\right) + \frac{D}{2}\mathrm{e}^{2x}\right)$

dM1: This requires

- the limits 4 and 0 must be substituted into an expression of the form $ax + b\sin 2x + c\mathrm{e}^x\left(\cos x \pm \sin x\right) + d\mathrm{e}^{2x}$ either way around and subtracted. This may be implied by a decimal answer which you may need to check.
- the answer to the above multiplied by 2π (An appreciation that the shape consists of two solids)

A1: awrt 290 cm^3 (condone lack of units).

If they have correct integration, correct limits followed by awrt 290, the last two marks can be awarded. Note that an answer of 460 is achieved when candidates are working in degrees and scores the method mark but not the accuracy mark.

Question Number	Scheme	Marks
10	Assume there is an angle x, $90° < x < 180°$, $\left\|\dfrac{\cos 2x}{\cos x - \sin x}\right\| \ldots 1$	B1
	$\dfrac{\cos 2x}{\cos x - \sin x} \equiv \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x - \sin x}$	M1
	$\lvert \cos x + \sin x \rvert \ldots 1 \Rightarrow (\cos x + \sin x)^2 \ldots 1 \Rightarrow 1 + 2\sin x \cos x \ldots 1 \Rightarrow \sin 2x \ldots 0$	dM1
	This is a contradiction, as sin α is negative for all angles $180° < \alpha < 360°$, hence true *	A1*
		(4 marks)

Condone strict inequalities for the first 3 marks in this question

B1: Sets up the argument.

It requires

- words like 'assume there is an angle x' or 'let there be an angle x', condoning 'for all angles x'
- the domain '$90° < x < 180°$'
- the contradiction $\left|\dfrac{\cos 2x}{\cos x - \sin x}\right| \ldots 1$ but condone $\left|\dfrac{\cos 2x}{\cos x - \sin x}\right| > 1$

M1: Uses the double angle formula for cos 2x and simplifies $\dfrac{\cos 2x}{\cos x - \sin x}$ to $(\cos x + \sin x)$

It may be scored when prematurely removing the modulus before proceeding to $(\cos x + \sin x) \ldots 1$

dM1: Squares $|(\cos x + \sin x)| \ldots 1$ and uses suitable identities in proceeding to $\sin 2x \ldots 0$, condoning $\sin 2x > 0$

It must be in a form in which the contradiction can easily be proven

A1: Deduces contradiction from opening statement and concludes. May consider the signs of sin x and cos x in the given range and deduce that the product is negative leading to the contradiction of the original assumption. Withold this mark if the modulus signs are not correctly dealt with. This mark is withheld for using strict inequalities

There will be many alternative approaches so you will need to look at what they have done to see if there is any credit worthy work. If in doubt please use review.

Alternative using *R*-alpha method

B1 M1: As above

dM1: Proceeds to the form $\left|\sqrt{2}\cos(x - 45°)\right| \ldots 1$ with the modulus seen

Alternatively, to the form $\left|\sqrt{2}\sin(x + 45°)\right| \ldots 1$ with the modulus seen

A1*: Considers $x \in (90°, 180°) \Rightarrow (x - 45°) \in (45°, 135°)$

Deduces that $|\cos(x - 45°)| < \dfrac{1}{\sqrt{2}}$ which means that $\left|\sqrt{2}\cos(x - 45°)\right| < 1$ so meaning that the given statement is true.

Alternatively