Do **not** allow as coordinates and do **not** allow $\begin{pmatrix} 12\mathbf{i} \\ 6\mathbf{j} \\ -3\mathbf{k} \end{pmatrix}$ but apply isw once a correct vector is seen.

(c)

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to find the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$
This may be implied by their value so you may need to check.
If the value is incorrect and no method is shown score M0

ddM1: Full attempt to find angle CAB using the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$

A1: $\theta =$ awrt $33.7°$ and no other angles (Degrees symbol **not** required)

Note that in (a) they can use any multiples of $\pm\overrightarrow{AC}$ and $\pm\overrightarrow{AB}$ to find the required angle.

Alternative:

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around **and** attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OB}$ either way around and attempts the lengths or lengths^2 of AB, BC and AC

ddM1: Full attempt to find angle CAB using the cosine rule

A1: $\theta =$ awrt $33.7°$ and no other angles (Degrees symbol **not** required)

(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

(d)

M1: Attempts a complete and correct method for finding one position for D.
(May be implied by at least 2 correct or correct ft components)

Examples:

Starting from $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$: $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from $A\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix}$: $\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} + 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} - 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from $B\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix}$: $\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 9\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **or** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

Starting from $A\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **or** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from $B\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **or** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: One correct point $(22, 9, -2)$ or $(-26, -15, 10)$

Condone if given as a vector e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ or $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

dM1: Attempts a complete and correct method for finding both possible positions for *D*.
(May be implied by at least 2 correct or correct ft components)

Examples:

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+5\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-7\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from $A\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+6\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-6\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from $B\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+3\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-9\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

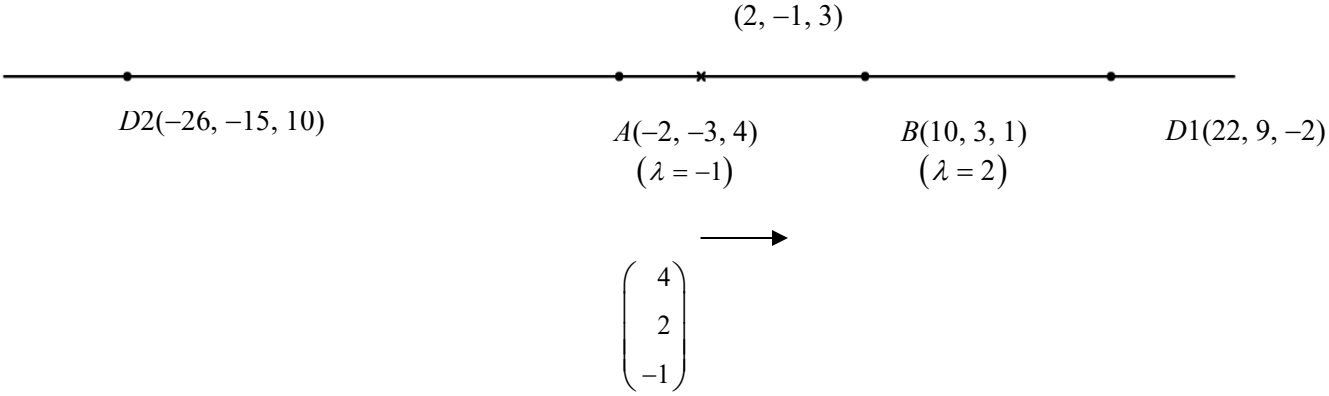
Starting from $A\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from $B\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: Gives both possible coordinates $(-26, -15, 10)$ and $(22, 9, -2)$

Condone if given as vectors e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ and $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

Configuration in (d):



Note that there may be more convoluted methods in (d) e.g.

$$\text{Area } CAD = 2 \times \text{Area } CAB \Rightarrow AD = 2\sqrt{12^2 + 6^2 + 3^2} \Rightarrow AD^2 = 4 \times 189$$

$$\overrightarrow{AD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4\lambda + 4 \\ 2\lambda + 2 \\ -\lambda - 1 \end{pmatrix}$$

$$(4\lambda + 4)^2 + (2\lambda + 2)^2 + (-\lambda - 1)^2 = 756$$

$$\Rightarrow 21\lambda^2 + 42\lambda - 735 = 0 \Rightarrow \lambda = 5, -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \textbf{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

In such cases, marks can be awarded as above for:

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

If you are in any doubt whether a method is sound or not use review.

Can also be done via the area e.g.

$$\text{Area } CAB = \frac{1}{2} AB \times AC \sin\theta = \frac{1}{2}\sqrt{189}\sqrt{172}\sin\theta$$

$$\cos\theta = \frac{25}{\sqrt{43}\sqrt{21}} \Rightarrow \sin\theta = \sqrt{\frac{278}{903}}$$

$$\text{Area } CAD = \frac{1}{2} AD \times AC \sin\theta = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\overrightarrow{AD} = \begin{pmatrix} 4\lambda+4 \\ 2\lambda+2 \\ -\lambda-1 \end{pmatrix} \therefore \frac{1}{2}\sqrt{(4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2}\sqrt{172}\sqrt{\frac{278}{903}} = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\Rightarrow (4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2 = 756$$

$$\Rightarrow 21\lambda^2+42\lambda-735=0 \Rightarrow \lambda = 5, -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \textbf{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

The same marking principles apply

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

For this method it is unlikely that candidates will work in exact terms and will revert to decimals.
This is acceptable but is unlikely to result in correct exact coordinates but the method marks are available as long as the method is complete and correct.

If candidates do work in decimals and then round their inexact values for the coordinates to the correct ones the A marks should be withheld.

Question Number	Scheme	Marks
9(a)(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\dfrac{3\sin^2 t\cos t}{-4\sin 2t}$	M1A1
	$\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{3\sin^2 t\cos t}{-8\sin t\cos t}=-\dfrac{3}{8}\sin t$ or e.g. $\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{\frac{3}{2}\sin 2t\sin t}{-4\sin 2t}=-\dfrac{3}{8}\sin t$	A1
(ii)	$t=\dfrac{\pi}{6}\quad\Rightarrow\quad x=1,\quad y=\dfrac{1}{8}$	B1
	$m=-\dfrac{3}{8}\sin\left(\dfrac{\pi}{6}\right)=-\dfrac{3}{16}\ \Rightarrow y-\dfrac{1}{8}=-\dfrac{3}{16}(x-1)$	M1
	$3x+16y-5=0$ *	A1*
		(6)
(b)	$3(2\cos 2t)+16(\sin^3 t)-5=0$	M1
	$6(1-2\sin^2 t)+16\sin^3 t-5=0$	dM1
	$16\sin^3 t-12\sin^2 t+1=0$	A1
	$(2\sin t-1)^2(4\sin t+1)=0\Rightarrow\sin t=-\dfrac{1}{4}$	ddM1
	$\sin t=-\dfrac{1}{4}\Rightarrow x=2\cos 2t=..$ **and** $y=\sin^3 t=...$	dddM1
	$Q\left(\dfrac{7}{4},-\dfrac{1}{64}\right)$	A1
		(6)
		(12 marks)

Notes:

(a)(i) **Note that (a)(i) is now being marked as M1A1A1 not M1dM1A1**

M1: Attempts to use the rule $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$. Condone incorrect attempts on $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t}$

Note that x and/or y may have been adapted before differentiation.

A1: Correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in any form **from correct work.**

A1: Achieves $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{3}{8}\sin t$ **<u>from fully correct work so must follow M1A1 so M1A0A1 is not possible.</u>**

You may need to check the working carefully as the value of k can be deduced from the given tangent equation.

e.g. $3x+16y-5=0\Rightarrow m=-\dfrac{3}{16},\ m=k\sin\dfrac{\pi}{6}\Rightarrow k=-\dfrac{3}{8}$