Question Number	Scheme	Marks
7	$x = 4\sin\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 4\cos\theta$	B1
	$\displaystyle\int \frac{1}{\left(16-x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x = \int \frac{4\cos\theta}{\left(16-16\sin^2\theta\right)^{\frac{3}{2}}}\,\mathrm{d}\theta = \int \frac{4\cos\theta}{64\cos^3\theta}\,\mathrm{d}\theta$	M1 A1
	$= \dfrac{1}{16}\tan\theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $\mathrm{f}\left(\frac{\pi}{3}\right)-\mathrm{f}\left(\frac{\pi}{6}\right)$ or $\mathrm{f}(60°)-\mathrm{f}(30°)$	M1
	$= \dfrac{1}{16}\tan\dfrac{\pi}{3} - \dfrac{1}{16}\tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{24}$	A1
		(6 marks)

Notes:

B1: States or uses $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 4\cos\theta$ o.e. e.g. $\mathrm{d}x = 4\cos\theta\mathrm{d}\theta$. This may be seen within the integrand.

M1: Simplifies $\left(16-x^2\right)^{\frac{3}{2}}$ using the given substitution to $k\cos^3\theta$ using the Pythagorean identity

This may be implied if the work is correct and they proceed correctly to e.g. $\dfrac{1}{16}\displaystyle\int \cos^{-2}\theta\ \mathrm{d}\theta$

A1: $\displaystyle\int \frac{1}{\left(16-x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x = \int \frac{4\cos\theta}{64\cos^3\theta}\,\mathrm{d}\theta$ or exact equivalent e.g. $\dfrac{1}{16}\displaystyle\int \cos^{-2}\theta\ \mathrm{d}\theta$ or $\displaystyle\int \frac{1}{\left(4\cos\theta\right)^2}\mathrm{d}\theta$

or e.g. $\displaystyle\int \frac{4\cos\theta}{\left(4\cos\theta\right)\left(4\cos\theta\right)\left(4\cos\theta\right)}\,\mathrm{d}\theta$

Condone if the "dθ" is missing but is otherwise correct.

dM1: For $\displaystyle\int k\frac{\cos\theta}{\cos^3\theta}\mathrm{d}\theta = \ldots\tan\theta$

M1: Uses limits of $\frac{\pi}{6}(30°)$ and $\frac{\pi}{3}(60°)$ correctly with their attempted integration.

Condone poor integration here.

A1: $\dfrac{\sqrt{3}}{24}$ or exact equivalent e.g. $\dfrac{1}{8\sqrt{3}}$

Question Number	Scheme	Marks
8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \ldots$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\,\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \lvert AC \rvert \lvert AB \rvert \cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos\theta \Rightarrow \theta = \ldots$ $\left(NB \quad \sqrt{172} = 2\sqrt{43},\ \sqrt{189} = 3\sqrt{21} \right)$	ddM1
	$\theta =$ awrt $33.7°$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2,\ BC^2 = 6^2 + 4^2 + 3^2,\ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC\cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172}\cos\theta \Rightarrow \cos\theta = \ldots$	ddM1
	$\theta =$ awrt $33.7°$	A1
		(4)

(a)

B1: States or implies that $a = -3$ **or** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

B1: States or implies that $a = -3$ **and** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

In the rest of the question, you can condone slips in writing down vectors as long as the intention is clear.

(b)

M1: Attempts to subtract their vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way round.
If no method is shown, it can be implied by at least 2 correct components for their vectors.

A1: Correct vector using correct notation.

Do **not** allow as coordinates and do **not** allow $\begin{pmatrix} 12\mathbf{i} \\ 6\mathbf{j} \\ -3\mathbf{k} \end{pmatrix}$ but apply isw once a correct vector is seen.

(c)

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to find the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$
This may be implied by their value so you may need to check.
If the value is incorrect and no method is shown score M0

ddM1: Full attempt to find angle CAB using the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$

A1: $\theta =$ awrt $33.7°$ and no other angles (Degrees symbol **not** required)

Note that in (a) they can use any multiples of $\pm\overrightarrow{AC}$ and $\pm\overrightarrow{AB}$ to find the required angle.

Alternative:

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around **and** attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OB}$ either way around and attempts the lengths or lengths^2 of AB, BC and AC

ddM1: Full attempt to find angle CAB using the cosine rule

A1: $\theta =$ awrt $33.7°$ and no other angles (Degrees symbol **not** required)

(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

(d)

M1: Attempts a complete and correct method for finding one position for D.
(May be implied by at least 2 correct or correct ft components)

Examples:

Starting from $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$: $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from $A\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix}$: $\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} + 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} - 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from $B\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix}$: $\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 9\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.