(i) Alternative:

B1: States or uses $V = 70\pi t$

B1: States or uses $\frac{4}{3}\pi r^3 = 70\pi t$

M1: For either:

- Differentiating $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $...r^2 \frac{\mathrm{d}r}{\mathrm{d}t} = ...$ and substituting $r = 5$ **or**
- Making r the subject from $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $r = ...t^{\frac{1}{3}}$ and then differentiating to obtain $\frac{\mathrm{d}r}{\mathrm{d}t} = ...t^{-\frac{2}{3}}$ and then substituting t obtained from $\frac{4}{3}\pi(5)^3 = 70\pi t$

A1: $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.7$ oe (Units are not required but if any are given they must be correct)

(ii)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3} \Rightarrow \int h^3 \,\mathrm{d}h = \int k \,\mathrm{d}t \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0,\ h = 4 \Rightarrow \frac{1}{4} \times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5,\ h = 6 \Rightarrow \frac{1}{4} \times 6^4 = k \times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = ...$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Notes:

M1: Separates variables and integrates both sides to obtain $...h^4 = kt(+c)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4 = kt(+c)$ or equivalent, with or without a constant of integration.

M1: Requires a constant of integration and using $t = 0,\ h = 4$ to find their c.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dM1: Uses $t = 5,\ h = 6$ to find k.

It is dependent upon the **previous** mark and having two correctly placed constants.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for substituting $h = 10$ into their equation and finding T (or t)

A1: Awrt 46.8 or exact e.g. $\frac{609}{13}, \frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "T" so just look for the correct value.

Units are not required but if any are given they must be correct.

See next pages for alternatives to (ii)

Alternative using definite integration:

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{k}{h^3}\Rightarrow\int h^3\,\mathrm{d}h=\int k\,\mathrm{d}t\Rightarrow\left[\frac{1}{4}h^4\right]_4^6=\left[kt\right]_0^5$$

$$324-64=5k\Rightarrow k=52$$

$$\left[\frac{1}{4}h^4\right]_4^{10}=\left[52t\right]_0^T\Rightarrow 2436=52T\Rightarrow T=46.8$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $...h^4=kt\left(+c\right)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4=kt\left(+c\right)$ or equivalent, with or without a constant of integration.

M1: Attempts $\left[\frac{1}{4}h^4\right]_4^6=\frac{1}{4}\times6^4-\frac{1}{4}\times4^4=260$ Condone poor integration of h^3.

dM1: Attempts $\left[kt\right]_0^5=5k$ and solves $\left[\frac{1}{4}h^4\right]_4^6=\left[kt\right]_0^5$ to find k.

It is dependent upon the **previous** mark.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for attempting $\left[\frac{1}{4}h^4\right]_4^{10}=\left[52t\right]_0^T$ to find T (or t)

A1: Awrt 46.8 or exact e.g $\frac{609}{13}$, $\frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "T" so just look for the correct value.

Units are not required but if any are given they must be correct.

Alternative when *k* is placed differently:

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{k}{h^3}\Rightarrow\int\frac{h^3}{k}\mathrm{d}h=\int\mathrm{d}t\Rightarrow\frac{h^4}{4k}=t+c$$

$$t=0,\ h=4\Rightarrow\frac{64}{k}=c\quad\text{or}\quad t=5,\ h=6\Rightarrow\frac{324}{k}=5+c$$

$$\frac{64}{k}=c,\ \frac{324}{k}=5+c\Rightarrow c=\frac{16}{13},\ k=52$$

$$\frac{h^4}{208}=t+\frac{16}{13}\Rightarrow\frac{10^4}{208}=t+\frac{16}{13}\Rightarrow T=\frac{609}{13}$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $...\frac{h^4}{k}=t\left(+c\right)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{h^4}{4k}=t\left(+c\right)$ or equivalent, with or without a constant of integration.

M1: Requires a constant of integration and using either $t=0,\ h=4$ or $t=5,\ h=6$ to obtain an equation connecting *c* and *k*.

Condone poor integration for this mark.

dM1: Uses **both** $t=0,\ h=4$ **and** $t=5,\ h=6$ to obtain 2 equations connecting *c* and *k* and solves simultaneously to obtain a value for *c* and a value for *k*.

You do not need to be concerned how they solve their equations as long as they obtain a value for *c* and a value for *k*.

It is dependent upon the **previous** mark and having two correctly placed constants.

Condone poor integration for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for substituting $h=10$ into their equation and finding *T* (or *t*)

A1: Awrt 46.8 or exact e.g $\frac{609}{13}$, $\frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "*T*" so just look for the correct value.

Units are not required but if any are given they must be correct.

Question Number	Scheme	Marks
5(i)	$\int x^2 \mathrm{e}^{4x}\,\mathrm{d}x = \frac{1}{4}x^2\mathrm{e}^{4x} - \int \frac{1}{2}x\,\mathrm{e}^{4x}\,\mathrm{d}x$	M1 A1
	$= \frac{1}{4}x^2\mathrm{e}^{4x} - \frac{1}{8}x\mathrm{e}^{4x} + \int \frac{1}{8}\mathrm{e}^{4x}\,\mathrm{d}x$	dM1
	$= \frac{1}{4}x^2\mathrm{e}^{4x} - \frac{1}{8}x\mathrm{e}^{4x} + \frac{1}{32}\mathrm{e}^{4x} + c$	A1
		(4)

Notes:

(i)

M1: Attempts to integrate by parts once the correct way around. The minus must be present (not +)

Look for $ax^2\mathrm{e}^{4x} - \int bx\,\mathrm{e}^{4x}\,\mathrm{d}x, \quad a,\ b > 0$. Condone the omission of the "dx".

A1: $\frac{1}{4}x^2\mathrm{e}^{4x} - \int \frac{1}{2}x\,\mathrm{e}^{4x}\,\mathrm{d}x$ oe e.g. $\frac{1}{4}x^2\mathrm{e}^{4x} - \int \frac{2}{4}x\,\mathrm{e}^{4x}\,\mathrm{d}x$

dM1: Integrates $\ldots\int x\,\mathrm{e}^{4x}\,\mathrm{d}x$ by parts the correct way around again. The second sign can be + or –

This may be seen in isolation so award for $\ldots\int x\,\mathrm{e}^{4x}\,\mathrm{d}x = \ldots x\mathrm{e}^{4x} - \ldots\int \mathrm{e}^{4x}\,\mathrm{d}x$

Condone the omission of the "dx".

A1: $\frac{1}{4}x^2\mathrm{e}^{4x} - \frac{1}{8}x\mathrm{e}^{4x} + \frac{1}{32}\mathrm{e}^{4x}\,(+c)$ with or without a constant of integration.

Allow equivalent simplified expressions e.g. $\mathrm{e}^{4x}\left(\frac{1}{4}x^2 - \frac{1}{8}x + \frac{1}{32}\right)(+c)$ and apply isw once a correct simplified answer is seen.

<u>Watch for "D and I" method:</u>

D	I
x^2	e^{4x}
$2x$	$\frac{1}{4}\mathrm{e}^{4x}$
2	$\frac{1}{16}\mathrm{e}^{4x}$
	$\frac{1}{64}\mathrm{e}^{4x}$

In these cases score M1dM1 for obtaining:

$px^2\mathrm{e}^{4x} \pm qx\mathrm{e}^{4x} \pm r\mathrm{e}^{4x}, \quad p, q, r > 0$

and then A1 for the **correct** first 2 terms $\frac{1}{4}x^2\mathrm{e}^{4x} - \frac{1}{8}x\mathrm{e}^{4x}$

and then A1 as in the main scheme.

Note that for this approach M1A1dM0 is not possible.

(ii)	$\dfrac{2x+11}{(2x+1)(2-x)} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(2-x)} \Rightarrow A = ..., B = ...$	M1
	$= \dfrac{4}{(2x+1)} + \dfrac{3}{(2-x)}$	A1
	$\displaystyle\int \dfrac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln\lvert 2x+1\rvert - 3\ln\lvert 2-x\rvert$	dM1 A1ft
	$\displaystyle\int_4^7 \dfrac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = (2\ln 15 - 3\ln 5) - (2\ln 9 - 3\ln 2)$	ddM1
	$= \ln\dfrac{8}{45}$	A1
		(6)
		(10 marks)

Notes:

(ii)

M1: Attempts partial fractions. Condone slips but should be of the correct form $\dfrac{A}{(2x+1)} + \dfrac{B}{(2-x)}$

A1: Correct **fractions** $\dfrac{4}{(2x+1)} + \dfrac{3}{(2-x)}$ seen.

It is not for correct "*A*" and "*B*" unless the correct fractions are seen or implied by later work.

dM1: Integrates to a correct form e.g. $...\ln\lvert 2x+1\rvert + ...\ln\lvert 2-x\rvert$ where … are non-zero constants.

Moduli are not required, brackets will suffice but condone missing brackets for this mark e.g.

$$\int \frac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln 2x+1 - 3\ln 2-x$$

A1ft: Correct integration but follow through on their *A* and *B*.

Moduli are not required, brackets will suffice <u>but must be present now</u> or implied by subsequent work e.g. when they substitute limits. Allow unsimplified e.g. $\dfrac{4}{2}\ln(2x+1) - 3\ln(2-x)$

ddM1: Applies the limits 4 and 7 to a function of the correct form e.g. $...\ln\lvert 2x+1\rvert + ...\ln\lvert 2-x\rvert$ or $...\ln(2x+1) + ...\ln(2-x)$ and subtracts the right way round. Brackets around the $2x + 1$ and $2 - x$ must be present or implied but condone missing brackets around the expression for the lower limit e.g. condone $\displaystyle\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln 15 - 3\ln 5 - 2\ln 9 - 3\ln 2$

and condone e.g. $\displaystyle\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln 15 - 3\ln(-5) - (2\ln 9 - 3\ln(-2))$

May be implied by e.g. $\left[4\ln(2x+1)\right]_4^7 + \left[3\ln(2-x)\right]_4^7 = 4\ln\dfrac{15}{9} + 3\ln\dfrac{5}{2}$

A1: $\ln\dfrac{8}{45}$ cao. Do **not** allow $\ln\left\lvert -\dfrac{8}{45}\right\rvert$.