9. (a) Express $\dfrac{1}{x(2x-1)}$ in partial fractions.

(2)

The height above ground, h metres, of a carriage on a fairground ride is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)$$

where t seconds is the time after the start of the ride.

Given that, at the start of the ride, the carriage is 2.5 m above ground,

(b) solve the differential equation to show that, according to the model,

$$h = \frac{5}{10 - 8\mathrm{e}^{k\sin\left(\frac{t}{10}\right)}}$$

where k is a constant to be found.

(6)

(c) Hence find, according to the model, the time taken for the carriage to reach its maximum height above ground for the 3rd time.
Give your answer to the nearest second.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 10 marks)

10.

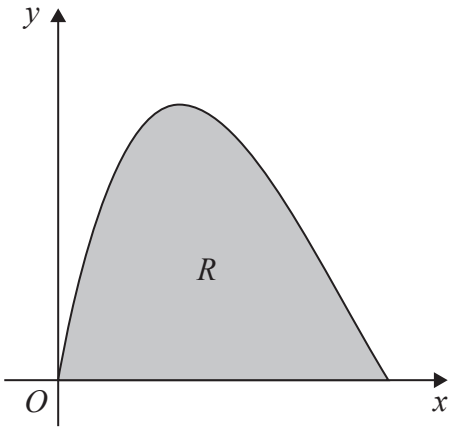

Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t\sin^2 t\cos t\,\mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

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