8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9. (a) Express $\dfrac{1}{x(2x-1)}$ in partial fractions.

(2)

The height above ground, h metres, of a carriage on a fairground ride is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)$$

where t seconds is the time after the start of the ride.

Given that, at the start of the ride, the carriage is 2.5 m above ground,

(b) solve the differential equation to show that, according to the model,

$$h = \frac{5}{10 - 8\mathrm{e}^{k\sin\left(\frac{t}{10}\right)}}$$

where k is a constant to be found.

(6)

(c) Hence find, according to the model, the time taken for the carriage to reach its maximum height above ground for the 3rd time.
Give your answer to the nearest second.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

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