Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h}=\frac{12}{30}\Rightarrow r=\frac{12}{30}h\Rightarrow V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{2}{5}h\right)^2h$	M1
	$V=\frac{4\pi h^3}{75}$*	A1*
		(2)
(b)	$V=\frac{4\pi h^3}{75}\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{12\pi h^2}{75}\left(=\frac{4\pi h^2}{25}\right)$	B1
	$V=1.5\times60\times2\pi$ $180\pi=\frac{4\pi h^3}{75}\Rightarrow h^3=3375\Rightarrow h=...(15)$	M1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{75}{12\pi\times15^2}\times2\pi=...$	M1
	$\frac{1}{18}$ (cm s^{-1})	A1
		(4)
		Total 6

(a)

M1: Uses the 12 and the 30 to find the ratio between *h* and *r* and substitutes into the volume formula to find *V* in terms of *h*. Condone poor bracketing / invisible brackets for this mark.

May just state $r=\frac{2}{5}h$

This mark cannot be for substituting in e.g. $r=\frac{2}{5}$ instead of $r=\frac{2}{5}h$

A1*: Correct proof with no errors including brackets. There must be at least one intermediate stage of working between establishing $r=\frac{12}{30}h$ and proceeding to the given answer.

(b) **Note that substituting $h=1.5$ or $h=90$ will score M0M0 in (b)**

B1: Correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ in any form

M1: Attempts their volume ("$2\pi\times t$") after 1.5 **minutes** and uses this with the given result from (a) to find *h*. They may find *h* by equating their expression for the volume to the given result from (a) before substituting $t=90$ and rearranging to find *h*. Do not be concerned by the mechanics of the rearrangement. Condone slips.

M1: Correct chain rule attempt using their *h* to find $\frac{\mathrm{d}h}{\mathrm{d}t}$. It is dependent on having used t = 90 (or condone t = 1.5) when finding *h*.

A1: Correct value. Allow awrt 0.0556 (cm s^{-1}) Units not required for this mark.

Alt(b) I Solving the differential equation $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{75}{6h^2}$ and then using $t=90$

B1: $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{12\pi h^2}{75}\left(=\frac{4\pi h^2}{25}\right)$

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}\left(=\frac{75}{6h^2}\right)$ to achieve an expression in *h*, attempts to solve their differential equation $\int 6h^2\,\mathrm{d}h=\int 75\,dt\Rightarrow 2h^3=75t+c$ o.e. and uses $t=0,\ h=0\Rightarrow c=0$

(you may not see the working for *c*) **to find *h* = f(*t*)** ($h=\left(37.5t\right)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}\left(=\frac{1}{3}\sqrt[3]{37.5}\,t^{-\frac{2}{3}}\right)$ and substitutes $t=90\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=...$

A1: As above in main scheme

Alt(b) II Equating volumes, rearranging to $h=...$ and then using $t=90$

B1: $V=2\pi t$

M1: Equates their $2\pi t$ equal to $\frac{4\pi h^3}{75}$ and rearranges to $h=\mathrm{f}(t)$ ($h=\left(37.5t\right)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}\left(=\frac{1}{3}\sqrt[3]{37.5}\,t^{-\frac{2}{3}}\right)$ and substitutes $t=90\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=...$

A1: As above in main scheme

Question Number	Scheme	Marks
6	$u=\sqrt{x^3+1} \Rightarrow u^2=x^3+1 \Rightarrow 2u\dfrac{\mathrm{d}u}{\mathrm{d}x}=3x^2$ or $u=\sqrt{x^3+1} \Rightarrow u^2=x^3+1 \Rightarrow 2u=3x^2\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or $u=\sqrt{x^3+1} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2}\left(x^3+1\right)^{-\frac{1}{2}}\times 3x^2$ or $x=(u^2-1)^{\frac{1}{3}} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\displaystyle\int \frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int \frac{9x^5}{u}\frac{2u}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$ or e.g. $\displaystyle\int \frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int \frac{9x^5}{\sqrt{x^3+1}}\frac{2\sqrt{x^3+1}}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$	M1A1
	$6\displaystyle\int\left(u^2-1\right)\mathrm{d}u=2u^3-6u\ (+c)=2\left(x^3+1\right)^{\frac{3}{2}}-6\left(x^3+1\right)^{\frac{1}{2}}\ (+c)$	M1
	$=2\left(x^3+1\right)^{\frac{1}{2}}\left[\left(x^3+1\right)-3\right]+c=2\left(x^3+1\right)^{\frac{1}{2}}\left(x^3-2\right)+c$	A1
		(5)
		Total 5

B1: Any correct equation involving $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or du and dx separately. May be implied by further work.

M1: Attempts to fully change $\displaystyle\int \frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x$ into an integral with respect to u. There must be no x's anywhere. Must include an attempt at replacing dx to get du so M0 if there is no du term present (unless implied by further work) or dx becomes du without an attempt at connecting them first. (i.e. there must have been an attempt at $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}$). Condone no integral sign for this mark.

A1: Reaches $6\displaystyle\int\left(u^2-1\right)\mathrm{d}u$ o.e. Condone the omission of the integral sign but they must have a du or implied by further work.

M1: Attempts to integrate their expression in terms of u only (usually $\int(Au^2+B)\,\mathrm{d}u$) **and** attempts to reverse the substitution so that the expression is completely in terms of x (with or without the constant of integration). For the integration look for a correct method to integrate a term i.e. $...u^n \rightarrow ...u^{n+1}$ or $B \rightarrow Bu$ or $\dfrac{C}{u} \rightarrow \ln u$ at least once. Just seeing + c is not an example of integrating.

A1: Correct expression in the required form including the + c