Question Number	Scheme	Marks
4(a)	$x=1, y=2 \Rightarrow 3x^2+2y^2-4xy+8^x-11=3+8-8+8-11=0$*	B1*
		(1)
(b)	$8^x \rightarrow 8^x \ln 8$	B1
	$4xy \rightarrow 4x\frac{dy}{dx}+4y$ or $y^2 \rightarrow 2y\frac{dy}{dx}$	M1
	$6x+4y\frac{dy}{dx}-4y-4x\frac{dy}{dx}+8^x \ln 8=0$ o.e.	A1
	$\frac{dy}{dx}(4y-4x)=4y-6x-8^x \ln 8 \Rightarrow \frac{dy}{dx}=...$	M1
	$\frac{dy}{dx}=\frac{4y-6x-8^x \ln 8}{4y-4x}$ o.e. e.g $\frac{dy}{dx}=\frac{6x+8^x \ln 8-4y}{4x-4y}$ or e.g $\frac{dy}{dx}=\frac{4y-6x-e^{x\ln 8} \ln 8}{4y-4x}$	A1
		(5)
(c)	$(1, 2) \rightarrow \frac{dy}{dx}=\frac{8-6-8\ln 8}{8-4} \Rightarrow y-2=\frac{4}{8\ln 8-2}(x-1)$	M1
	$y=0 \Rightarrow 0-2=\frac{4}{8\ln 8-2}(x-1) \Rightarrow x=...$	M1
	$x=2-12\ln 2$	A1
		(3)
		Total 9

(a)

B1*: Substitutes $x = 1$ and $y = 2$ and obtains 0 with no errors.
Minimum required without evaluating each term e.g. $3\times1+2\times2^2-4\times1\times2+8^1-11=0$. We just need a correct equation which evaluates as 0.
Alternatively, substitutes in $x = 1$, forms a quadratic in y i.e. $2y^2-4y=0$ o.e. and solves to find y via any method:
$3+2y^2-4y+8-11=0 \Rightarrow 2y^2-4y=0 \Rightarrow y=2$ (ignore the other solution 0)
They may at this stage substitute in $y = 2$ to get 0.
In any of the approaches a conclusion is not required.

(b)

B1: Correct differentiation of 8^x. Award for sight of $\ln 8\times8^x$ or $e^{x\ln 8}\times\ln 8$. Do not be concerned with the use of modulus signs around ln8

M1: For $y^2 \rightarrow ky\frac{dy}{dx}$ or $4xy \rightarrow Ax\frac{dy}{dx}+By$

A1: Fully correct differentiation. Condone the omission of = 0 or may be implied by further work. Accept $6x+4y\frac{dy}{dx}-4y-4x\frac{dy}{dx}+e^{x\ln 8}\ln 8=0$

Ignore if they start their differentiated line with e.g. $\frac{dy}{dx}=$

M1: Attempts to make $\frac{dy}{dx}$ the subject with **2 terms** in $\frac{dy}{dx}$ coming from the correct places. i.e. they must have two terms (one of each) of the form $\alpha x\frac{dy}{dx}$ and $\beta y\frac{dy}{dx}$ where α, β are non-zero constants.

A1: Any correct expression for $\frac{dy}{dx}$. Allow $y' = ...$ o.e. isw after a correct answer is seen

Condone poor dividing lines which do not go quite far enough provided an incorrect method was not seen. E.g. $\frac{4y-6x-8^x \ln 8}{4y-4x}$

(c) **Note that an incorrect derivative in part (b) can score maximum 110 in part (c)**

M1: Attempts to use $x = 1$ and $y = 2$ in their $\frac{dy}{dx}$ to find the gradient at P (condone if this is a decimal) and attempts to form the equation of the normal at P using the negative reciprocal gradient. Condone one sign slip when substituting in the coordinates into the straight line equation.

They may set $y = 0$ at this stage such that $-2 = \frac{4}{8\ln 8 - 2}(x-1)$ which is acceptable.

If they use $y = mx + c$ they must proceed as far as $c = ...$

M1: States $y = 0$ or substitutes $y = 0$ into their straight-line equation using a changed gradient from their $\frac{dy}{dx}$ and rearranges to find x. Do not be concerned by the mechanics of their rearrangement.

A1: Correct expression in the required form

Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h}=\frac{12}{30}\Rightarrow r=\frac{12}{30}h\Rightarrow V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{2}{5}h\right)^2h$	M1
	$V=\frac{4\pi h^3}{75}$*	A1*
		(2)
(b)	$V=\frac{4\pi h^3}{75}\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{12\pi h^2}{75}\left(=\frac{4\pi h^2}{25}\right)$	B1
	$V=1.5\times60\times2\pi$ $180\pi=\frac{4\pi h^3}{75}\Rightarrow h^3=3375\Rightarrow h=...(15)$	M1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{75}{12\pi\times15^2}\times2\pi=...$	M1
	$\frac{1}{18}$ (cm s^{-1})	A1
		(4)
		Total 6

(a)

M1: Uses the 12 and the 30 to find the ratio between *h* and *r* and substitutes into the volume formula to find *V* in terms of *h*. Condone poor bracketing / invisible brackets for this mark.

May just state $r=\frac{2}{5}h$

This mark cannot be for substituting in e.g. $r=\frac{2}{5}$ instead of $r=\frac{2}{5}h$

A1*: Correct proof with no errors including brackets. There must be at least one intermediate stage of working between establishing $r=\frac{12}{30}h$ and proceeding to the given answer.

(b) **Note that substituting $h=1.5$ or $h=90$ will score M0M0 in (b)**

B1: Correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ in any form

M1: Attempts their volume ("$2\pi\times t$") after 1.5 **minutes** and uses this with the given result from (a) to find *h*. They may find *h* by equating their expression for the volume to the given result from (a) before substituting $t=90$ and rearranging to find *h*. Do not be concerned by the mechanics of the rearrangement. Condone slips.

M1: Correct chain rule attempt using their *h* to find $\frac{\mathrm{d}h}{\mathrm{d}t}$. It is dependent on having used t = 90 (or condone t = 1.5) when finding *h*.

A1: Correct value. Allow awrt 0.0556 (cm s^{-1}) Units not required for this mark.
