Question Number	Scheme	Marks
3(a)	$x = 3\sin^3\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = 9\sin^2\theta\cos\theta$ o.e.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2\sin 2\theta}{9\sin^2\theta\cos\theta}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4\sin\theta\cos\theta}{9\sin^2\theta\cos\theta} = -\frac{4}{9}\mathrm{cosec}\,\theta$	A1
		(3)
(b)	$\theta = \frac{\pi}{6} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = "-\frac{8}{9}",\ x = 3\left(\frac{1}{2}\right)^3 = ...,\ y = 1+\frac{1}{2} = ...$	M1
	$y - \frac{3}{2} = -\frac{8}{9}\left(x - \frac{3}{8}\right)$	dM1
	$16x + 18y - 33 = 0$	A1
		(3)
(c)	e.g. $x = 3\sin^3\theta \Rightarrow \sin\theta = \sqrt[3]{\frac{x}{3}} \Rightarrow\ y = 1+\cos 2\theta = 1+1-2\sin^2\theta = 2-2\left(\frac{x}{3}\right)^{\frac{2}{3}}$	M1
	$y = 2-2\left(\frac{x}{3}\right)^{\frac{2}{3}} \Rightarrow 2\left(\frac{x}{3}\right)^{\frac{2}{3}} = 2-y \Rightarrow 8x^2 = 9(2-y)^3$*	A1*
	$q = 3$	B1
		(3)
Alt(c)	e.g. $8x^2 = 72\sin^6\theta,\quad 9(2-y)^3 = 9(2-\cos 2\theta - 1)^3 = 9\left(1-\left(1-2\sin^2\theta\right)\right)^3$	M1
	$72\sin^6\theta = 9\times 8\sin^6\theta \Rightarrow 8x^2 = 9(2-y)^3$*	A1*
	$q = 3$	B1
		Total 9

(a)

B1: Correct expression for $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ which may be part of their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Condone poor labelling/notation provided the intention is clear. Must be in terms of θ

May use identities before differentiating using the product rule e.g. $\sin\theta\left(\frac{1-\cos 2\theta}{2}\right)$

M1: Correct method for $\frac{\mathrm{d}y}{\mathrm{d}x}$. They must be attempting $\frac{\mathrm{d}y}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{d}x}$ o.e. condoning copying slips and condoning poor differentiation, labelling /notation provided the intention is clear.

A1: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4}{9}\mathrm{cosec}\,\theta$ with no errors but condone invisible brackets and poor notation being recovered. Allow equivalent fractions to $-\frac{4}{9}$. Note $\frac{\mathrm{d}y}{\mathrm{d}x} =$ may be seen in an earlier line of their solution which is acceptable.

Alt (a) Implicit differentiation – note that some candidates may find the cartesian equation and differentiate implicitly or explicitly.

B1: A correctly differentiated cartesian equation. (Do not be concerned with where the cartesian equation has come from for this mark)

e.g. $16x = 9\times 3\times(-1)\times(2-y)^2\times\dfrac{\mathrm{d}y}{\mathrm{d}x}$

M1: Substitutes in for x and y into their differentiated equation and rearranges to achieve an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. Condone poor differentiation.

A1: Achieves $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{4}{9}\mathrm{cosec}\,\theta$ with no errors but condone invisible brackets and poor notation being recovered. Allow equivalent fractions to $-\dfrac{4}{9}$. They must have shown correctly how they found the cartesian equation. i.e. they cannot just use the given result in (c).

(b)

M1: Attempts to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, x and y when $\theta = \dfrac{\pi}{6}$. Must achieve values for all three which may be implied by further work. Condone slips but if values are only stated then their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be correct for their k (look for $2\times$ their k) and one of x or y must be correct.

Note that they may use their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y $\left(="-\dfrac{16x}{27(2-y)^2}"\right)$ by either finding the differentiated equation in (a) or by converting in (b).

dM1: Complete method for the tangent using their values. It is dependent on the previous method mark. Condone one sign error when substituting in the coordinates. If they use $y = mx + c$ they must proceed as far as $c = ...$

A1: $16x + 18y - 33 = 0$ or any integer multiple of this equation. Must have = 0.

(c)

M1: A complete method to eliminate θ. Any identities used must be correct when substituted in. Do not allow the parametric equations to be expressed as $\theta =$ and equated. May also be awarded if seen in (a).

A1*: Correct proof with no errors including brackets although condone missing trailing brackets if it does not affect the expression. Must be seen in (c).

B1: Correct value of q. Do not accept $x = 3$ Can be scored for the domain $-3 \le x \le 3$ which must be in terms of x

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Alt(c)

M1: A complete method to express lhs and rhs both in terms of sin θ only or both in terms cos 2θ only. Any identities used must be correct when substituted in. They cannot proceed from $9(2-y)^3 = 72\sin^6\theta$ without an intermediate stage but condone $8x^2 = 72\sin^6\theta$

A1*: Fully correct work showing lhs = rhs and gives a conclusion. There should be no errors including brackets but condone missing trailing brackets if it does not affect the expression.

B1: As above in the notes for (c)

Question Number	Scheme	Marks
4(a)	$x=1, y=2 \Rightarrow 3x^2+2y^2-4xy+8^x-11=3+8-8+8-11=0$*	B1*
		(1)
(b)	$8^x \rightarrow 8^x \ln 8$	B1
	$4xy \rightarrow 4x\frac{dy}{dx}+4y$ or $y^2 \rightarrow 2y\frac{dy}{dx}$	M1
	$6x+4y\frac{dy}{dx}-4y-4x\frac{dy}{dx}+8^x \ln 8=0$ o.e.	A1
	$\frac{dy}{dx}(4y-4x)=4y-6x-8^x \ln 8 \Rightarrow \frac{dy}{dx}=...$	M1
	$\frac{dy}{dx}=\frac{4y-6x-8^x \ln 8}{4y-4x}$ o.e. e.g $\frac{dy}{dx}=\frac{6x+8^x \ln 8-4y}{4x-4y}$ or e.g $\frac{dy}{dx}=\frac{4y-6x-e^{x\ln 8} \ln 8}{4y-4x}$	A1
		(5)
(c)	$(1, 2) \rightarrow \frac{dy}{dx}=\frac{8-6-8\ln 8}{8-4} \Rightarrow y-2=\frac{4}{8\ln 8-2}(x-1)$	M1
	$y=0 \Rightarrow 0-2=\frac{4}{8\ln 8-2}(x-1) \Rightarrow x=...$	M1
	$x=2-12\ln 2$	A1
		(3)
		Total 9

(a)

B1*: Substitutes $x = 1$ and $y = 2$ and obtains 0 with no errors.
Minimum required without evaluating each term e.g. $3\times1+2\times2^2-4\times1\times2+8^1-11=0$. We just need a correct equation which evaluates as 0.
Alternatively, substitutes in $x = 1$, forms a quadratic in y i.e. $2y^2-4y=0$ o.e. and solves to find y via any method:
$3+2y^2-4y+8-11=0 \Rightarrow 2y^2-4y=0 \Rightarrow y=2$ (ignore the other solution 0)
They may at this stage substitute in $y = 2$ to get 0.
In any of the approaches a conclusion is not required.

(b)

B1: Correct differentiation of 8^x. Award for sight of $\ln 8\times8^x$ or $e^{x\ln 8}\times\ln 8$. Do not be concerned with the use of modulus signs around ln8

M1: For $y^2 \rightarrow ky\frac{dy}{dx}$ or $4xy \rightarrow Ax\frac{dy}{dx}+By$

A1: Fully correct differentiation. Condone the omission of = 0 or may be implied by further work. Accept $6x+4y\frac{dy}{dx}-4y-4x\frac{dy}{dx}+e^{x\ln 8}\ln 8=0$

Ignore if they start their differentiated line with e.g. $\frac{dy}{dx}=$