Question Number	Scheme	Marks
9(a)	$\dfrac{1}{x(2x-1)} \equiv \dfrac{A}{x} + \dfrac{B}{2x-1} \Rightarrow 1 \equiv A(2x-1) + Bx \Rightarrow A = ... \text{or } B = ...$	M1
	$\dfrac{1}{x(2x-1)} \equiv \dfrac{2}{2x-1} - \dfrac{1}{x}$	A1
		(2)
(b)	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{1}{50}h(2h-1)\cos\left(\dfrac{t}{10}\right) \Rightarrow \displaystyle\int \dfrac{1}{h(2h-1)}\mathrm{d}h = \int \dfrac{1}{50}\cos\left(\dfrac{t}{10}\right)\mathrm{d}t$	M1
	$\displaystyle\int \dfrac{1}{h(2h-1)}\mathrm{d}h = \int \left(\dfrac{2}{2h-1} - \dfrac{1}{h}\right)\mathrm{d}h = \ln(2h-1) - \ln h$	M1
	$\displaystyle\int \left(\dfrac{2}{2h-1} - \dfrac{1}{h}\right)\mathrm{d}h = \int \dfrac{1}{50}\cos\left(\dfrac{t}{10}\right)\mathrm{d}t \Rightarrow \ln(2h-1) - \ln h = \dfrac{1}{5}\sin\left(\dfrac{t}{10}\right)\ (+c)$	A1ft
	e.g. $t = 0, h = 2.5 \Rightarrow \ln(2\times 2.5 - 1) - \ln 2.5 = c$ or e.g. $\ln\dfrac{2h-1}{h} = \dfrac{1}{5}\sin\left(\dfrac{t}{10}\right) + c \Rightarrow \dfrac{2h-1}{h} = X\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}$ $\quad t = 0, h = 2.5 \Rightarrow X = 1.6$	M1
	$\ln(2h-1) - \ln h = \dfrac{1}{5}\sin\left(\dfrac{t}{10}\right) + \ln 1.6 \Rightarrow \dfrac{2h-1}{h} = 1.6\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}$ $\Rightarrow 2h - 1.6h\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)} = 1 \Rightarrow h = ...$	M1
	$h = \dfrac{5}{10 - 8\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}}$	A1
		(6)
(c)	$\sin\left(\dfrac{t}{10}\right) = 1 \Rightarrow \dfrac{t}{10} = \dfrac{\pi}{2}, \dfrac{5\pi}{2}, \dfrac{9\pi}{2}, ... \Rightarrow t = ...$	M1
	$t = 45\pi = 141$ (s)	A1
		(2)
		Total 10

(a)

M1: Correct attempt at partial fractions. Sets $\dfrac{1}{x(2x-1)} = \dfrac{A}{x} + \dfrac{B}{2x-1}$ and uses a correct method to identify at least one constant. Incorrect work e.g. $\dfrac{A}{x} + \dfrac{B}{2x-1} \Rightarrow 1 = Ax + B(2x-1)$ etc. is M0.

A1: Correct partial fractions. Note that this mark is not just for the correct constants, it is for the correct written fractions but allow this mark to be scored if it is seen in part (b) (in terms of h or x).

Condone $\dfrac{2}{2x-1} + \dfrac{-1}{x}$ or written as e.g. $2(2x-1)^{-1} - x^{-1}$

(b) **Note that they may work in *x* throughout, but the final answer must be** $h=...$

M1: Attempts to separate the variables. Do not be concerned by slips with the constants so look for as a minimum $\int \frac{1}{h(2h-1)} = \int \cos(...t)$ o.e. May omit an integral sign on one of the sides or may be implied by their integrated expression.

M1: For $\int \left(\frac{A}{h}+\frac{B}{2h-1}\right) \mathrm{d}h = \alpha \ln h + \beta \ln(2h-1)$

or e.g. $\lambda \int \left(\frac{"A"}{h}+\frac{"B"}{2h-1}\right) \mathrm{d}h = \alpha \ln h + \beta \ln(2h-1)$ (if they attempt some rearranging first)

Condone invisible brackets.

A1ft: Fully correct equation following through their *A* and *B*. The "+*c*" is not required here.

$\int \left(\frac{A}{h}+\frac{B}{2h-1}\right)\mathrm{d}h = \int \frac{1}{50}\cos\left(\frac{t}{10}\right)\mathrm{d}t \Rightarrow A\ln h + \frac{B}{2}\ln(2h-1) = \frac{1}{5}\sin\left(\frac{t}{10}\right)$ $(+c)$ o.e.

Condone invisible brackets.

M1: States or uses $t = 0$, $h = 2.5$ followed by $c=...$ is sufficient for M1. You do not need to check their substitution/rearrangement but if there is just a value for the constant then you may need to check they used $t = 0$, $h = 2.5$. May be implied by e.g. $\ln 8 - \ln 5$, $\ln 1.6$, awrt 0.47 following correct integration. Note that they may have removed lns first.

M1: To score this mark they must have integrated and achieved an expression of the form

$$\pm p\ln h \pm q\ln(2h-1) = \pm r\sin\left(\frac{t}{10}\right)+c \text{ o.e.}$$

Rearranges using correct algebra to make *h* the subject including when combining logs and when dealing with the constant. May occur before finding their constant but there must have been a constant.

Expect to see terms in *h* collected on one side as part of their manipulation rather than proceeding to the given answer once they have established the value of *k*. Alternatively, they may cancel e.g. $\frac{2h-1}{h} \to 2-\frac{1}{h}$ and rearrange to make *h* the subject using correct algebra.

Do not condone sign slips for this mark.

A1: Correct expression (including $h=...$) in the required form. Requires

- Correct integration
- Correct algebra

Do not penalise brackets recovered in their work or working in *x* rather than *h* until the final answer.

(c) **Note that an answer with no working seen scores M0A0**

M1: Solves $\sin\left(\frac{t}{10}\right)=1$ to find a value for *t*. Do not allow this mark to be implied by e.g. 5π, 25π, 45π but allow for e.g. $\frac{t}{10}=\frac{\pi}{2} \Rightarrow t=..$ Must be working in radians.

Note that attempts that do not use part (b) and just use the differential equation proceeding to $\cos\left(\frac{t}{10}\right)=0$ score M0A0. (**Note if their *k* is negative condone if they solve** $\sin\left(\frac{t}{10}\right)=-1$)

A1: awrt 141 (but do not allow just 45π) Units not required.

Question Number	Scheme	Marks
10(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 6t$ o.e.	B1
	$\text{Area} = \left(\int \sin t \sin 2t \times 6t \,\mathrm{d}t = \right) \int \sin t \times 2\sin t \cos t \times 6t \,\mathrm{d}t$	M1
	$= 12\int_0^{\frac{\pi}{2}} t\sin^2 t \cos t \,\mathrm{d}t$	A1
		(3)
(b)	$= (12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}} - (12)\int_0^{\frac{\pi}{2}} \frac{1}{3}\sin^3 t \,\mathrm{d}t$	M1
	$= (12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3}\right)\int_0^{\frac{\pi}{2}} \sin t\left(1-\cos^2 t\right)\mathrm{d}t$	M1
	$= \left[4t\sin^3 t + 4\cos t - \frac{4}{3}\cos^3 t\right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3}\right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8

(a) On EPEN this is M1M1A1 but we are marking this as B1M1A1

It is acceptable to work in a different variable such as θ instead of t provided the final answer is in terms of t but withhold the final mark if there is a mixture of variables in the same line of working.

B1: $\frac{\mathrm{d}x}{\mathrm{d}t} = 6t$ seen or implied.

$6t\sin t\sin 2t$ implies B1 but $6t\sin^2 t\cos t$ does not imply B1.

M1: Uses $\sin 2t = 2\sin t\cos t$ within the expression for y i.e. $y = \sin t\sin 2t = 2\sin^2 t\cos t$ or may be seen in their integral. May be implied by e.g. $\int pt\sin t\sin 2t\ (\mathrm{d}t) \to \int 2pt\sin^2 t\cos t\ (\mathrm{d}t)$

Condone the omission of t. i.e. $y = 2\sin^2 \cos$

A1: Correct integral including $\mathrm{d}t$ with $k = 12$ **with at least one intermediate stage of working** (which may only be e.g. $\frac{\mathrm{d}x}{\mathrm{d}t} = 6t$) The integral sign, dt, limits only need to be present in the final answer. Ignore any attempts to verify the limits of 0 and $\frac{\pi}{2}$.

Do not condone poor notation e.g. $\sin t^2$ or the absence of variables in the main body of the solution.

$12t\sin^2 t\cos t$ with or without integration notation and with no other working implies B1M1A0