Question	Scheme	Marks
8(a)	$(8-3x)^{\frac{4}{3}}=16\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ but condone $16(1-kx)^{\frac{4}{3}}, k\neq 3$	**M1**
	"Correct" term 3 or term 4 in $(1-kx)^{\frac{4}{3}}=1\pm\frac{4}{3}kx+\frac{\frac{4}{3}\times\frac{1}{3}}{2}(\pm kx)^2+\frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}(\pm kx)^3+...$	**M1**
	Two terms correct of $16-8x+\frac{x^2}{2}+\frac{x^3}{24}+...$ following M1, M1 and $k=\pm\frac{3}{8}$	**A1**
	$=16-8x+\frac{x^2}{2}+\frac{x^3}{24}+...$	**A1**
		(4)
(b)	Assume the curves meet so $8x-\frac{15x^2}{2}+8="16-8x+\frac{x^2}{2}+\frac{x^3}{24}"$ (for some x)	**M1**
	$\Rightarrow 0=8-16x+8x^2+\frac{x^3}{24}+...\Rightarrow 0=8\left(x^2-2x\right)+8+\frac{x^3}{24}+...\Rightarrow 8\left(x-1\right)^2\pm..+..$	**M1**
	$\Rightarrow 0=8\left(x-1\right)^2+\frac{x^3}{24}$	**A1**
	Requires • Argument that $8\left(x-1\right)^2+\frac{x^3}{24}>0$. E.g. $8\left(x-1\right)^2\geqslant 0$ and $\frac{x^3}{24}>0$ (for $0<x<\frac{8}{3}$) so $8\left(x-1\right)^2+\frac{x^3}{24}>0$ • Minimal statement (contradiction) and conclusion	**A1**
		(4)
		(8 marks)

(a)

M1: Attempts to takes out a factor of 16 or $8^{\frac{4}{3}}$. Look for $(8-3x)^{\frac{4}{3}}=16(1-kx)^{\frac{4}{3}},\quad k\neq 3$

or $(8-3x)^{\frac{4}{3}}=8^{\frac{4}{3}}(1-kx)^{\frac{4}{3}},\quad k\neq 3$ Condone incorrect values of k other than 3

M1: Attempts the binomial expansion on $\left(1-kx\right)^{\frac{4}{3}}$ achieves at least a "correct" term 3 or term 4 (which may be unsimplified) for their k. Even allow with $k=1$

Look for either $\frac{\frac{4}{3}\times\frac{1}{3}}{2}(\pm kx)^2$ or $\frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}(\pm kx)^3$ with the correct binomial coefficient and the correct power of x. Condone missing brackets on the $\left(kx\right)$ terms.

If they expand $\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ without showing the unsimplified terms it is implied by $\pm\frac{x^3}{384}$

A1: For two correct and simplified terms of $16-8x+\frac{x^2}{2}+\frac{x^3}{24}+...$ following M1, M1 and $k=\pm\frac{3}{8}$

A1: Correct expansion from correct work. Terms may be as a list for both accuracy marks

Alt (a) by direct expansion. $(8\pm 3x)^{\frac{4}{3}} = 8^{\frac{4}{3}} + \frac{4}{3}\left(8^{\frac{1}{3}}\right)\left(\pm 3x\right) + \frac{\frac{4}{3}\times\frac{1}{3}}{2}\left(8^{-\frac{2}{3}}\right)(\pm 3x)^2 + \frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}\left(8^{-\frac{5}{3}}\right)(\pm 3x)^3 + ...$

M1: For $8^{\frac{4}{3}}$ or 16 oe seen as the constant term E.g. $= 16 \pm ...x + ...$

M1: For the correct term 3 or term 4 of the expansion. Condone missing brackets on the $(\pm 3x)^2$, $(\pm 3x)^3$ terms

A1: For two correct and simplified terms of $16 - 8x + \frac{x^2}{2} + \frac{x^3}{24} + ...$ Allow as a list

A1: Correct expansion from correct work. Allow as a list

(b)

M1: Sets up the contradiction hypothesis using the result of their (a) and setting equal to the quadratic equation. There must be some words so look for a minimum of

Assume they meet (or simply let) $8x - \frac{15x^2}{2} + 8 = \text{"16"} - 8x + \frac{x^2}{2} + \frac{x^3}{\text{"24"}}$

M1: Rearranges terms and attempts to complete the square (or factorises as *k* (*x*-1) (*x*-1)) on the quadratic terms or other means to show the quadratic part is always positive (e.g. considers discriminant).

The work on the quadratic terms of the expression, which must be of the form $8x^2 - 16x + (A-8)$ should take the form of one of

- $8x^2 - 16x + (A-8) = 8\left(x-1\right)^2 + ...$
- $8x^2 - 16x + (A-8)$ and attempts $b^2 - 4ac = 16^2 - 4\times 8\left(A-8\right) = ...$

A1: Correctly completes the square to form equation $0 = 8\left(x-1\right)^2 + \frac{x^3}{24}$.

Via the discriminant you would need $b^2 - 4ac = 16^2 - 4\times 8\times 8 = 0$,

A1: Completes the argument by drawing together the contradiction.

States that $8\left(x-1\right)^2 \geqslant 0$ and $\frac{x^3}{24} > 0$ (in the interval), hence $0 \neq 8\left(x-1\right)^2 + \frac{x^3}{24}$ so proven

Stating that $8\left(x-1\right)^2 + \frac{x^3}{24} > 0$ is insufficient without some explanation. (See above)

Stating that both terms are positive (as $x > 0$) is not enough as $8\left(x-1\right)^2 = 0$ at $x = 1$

For discriminant must mention $b^2 - 4ac = 0$, hence single root (at 1) and positive quadratic or similar, $\frac{x^3}{24} > 0$ hence $0 \neq 8\left(x-1\right)^2 + \frac{x^3}{24}$ so proven.

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There will be alternatives for the middle two marks in (b), so consider carefully what is attempted.

For example, candidates could work on the cubic $8 - 16x + 8x^2 + \frac{x^3}{24}$. Condone use of calculus which would involve some calculator work that proves that the function has a minimum value at x = awrt 0.99 y = awrt 0.04 and hence $8 - 16x + 8x^2 + \frac{x^3}{24} \neq 0$ (for $0 < x < \frac{8}{3}$) proving that there is a contradiction. M1: Differentiates, sets = 0 and solves to find x and y in the interval. A1: Correct with proof of minimum

Note that merely solving $8 - 16x + 8x^2 + \frac{x^3}{24} = 0$ on a calculator to get $x = -194$ and stating this is not in the range is M0.

Question	Scheme	Marks
9(a)	Volume, $V = \pi\int_{\frac{1}{3}}^{3} y^2\,\mathrm{d}x = \pi\int_{\frac{1}{3}}^{3} \dfrac{x^{-\frac{1}{2}}}{(1+x)\left(\arctan\left(\sqrt{x}\right)\right)^2}\,\mathrm{d}x$	**B1**
	$\tan u = \sqrt{x} \Rightarrow \sec^2 u\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{2}x^{-\frac{1}{2}}$ oe	**M1** **A1**
	$\Rightarrow V = (\pi)\displaystyle\int \dfrac{2\times\frac{1}{2}x^{-\frac{1}{2}}}{\left(\arctan\left(\sqrt{x}\right)\right)^2(1+x)}\,\mathrm{d}x = (\pi)\int \dfrac{2}{u^2\left(1+\tan^2 u\right)}\sec^2 u\,\mathrm{d}u$	**dM1**
	$= \pi\displaystyle\int \dfrac{2}{u^2\cancel{\sec^2 u}}\cancel{\sec^2 u}\,\mathrm{d}u = 2\pi\int\dfrac{1}{u^2}\,\mathrm{d}u$	**A1**
	$\left.\begin{array}{l} x=3\Rightarrow u=\arctan\sqrt{3}=\dfrac{\pi}{3} \\ x=\dfrac{1}{3}\Rightarrow u=\arctan\dfrac{1}{\sqrt{3}}=\dfrac{\pi}{6}\end{array}\right\}\Rightarrow V=2\pi\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\dfrac{1}{u^2}\,\mathrm{d}u$	**B1**
		(6)
(b)	$V = (2\pi)\left[-\dfrac{1}{u}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	**M1**
	$= \left(2\pi\right)\left[-\dfrac{1}{\pi/3}-\left(-\dfrac{1}{\pi/6}\right)\right]$ Or $= (2\pi)\left[-\dfrac{1}{\arctan\sqrt{x}}\right]_{\frac{1}{3}}^{3} = (2\pi)\left(-\dfrac{1}{\arctan\sqrt{3}}-\left(-\dfrac{1}{\arctan\sqrt{1/3}}\right)\right)$	**dM1**
	$= 2\pi\left(-\dfrac{3}{\pi}+\dfrac{6}{\pi}\right) = 6$	**A1**
		(3)
		(9 marks)